Definition of Frequency of Exceedances of Structural Design Parameters

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1. Introduction

The design of an intake structure for supersonic combat aircraft is highly dependent on assumptions to be defined in the early design phase and in the check stress and structural clearance phases, see ref. 1
The assumptions to be made are related mainly to the dynamic hammershock pressure wave and its dynamic behavior in terms of magnitude depending on pressure at the engine face, the shape of the intake / duct, the flight condition, the change of magnitude during its travel from the air intake to the engine along the duct to the forward intake and the effect of surge interaction in case of two neighborhood engine intake / ducts. The assumptions have to be based on an existing project and a statistical approach has to be chosen with respect to the probability of occurrence of the hammershock for flight hours of each aircraft and the complete aircraft fleet as well as the aircraft missions. Beside investigation about uncertainty the effects of dynamic response of the intake duct structure has to be carefully estimated during design.

The design philosophy can be based on the concept that the structure is able to carry for the limit load case a static loading consisting of flight maneuver loads, steady state pressure and a maximum positive and negative hammershock pressure factorized by a dynamic factor and that the structure withstands ultimate loading resulting from steady state pressure and maneuver loads with allowance for plastic deformation due to ultimate hammershock pressure. A probability analysis should be performed to receive the numbers of exceedances of design parameters.
For both concepts it is essential during the different design and clearance phases to verify the assumptions made from the beginning using comparisons of different methods, experimental results from modal tests, on aircraft ground surge interaction tests and flight test results. Only careful consideration of all dynamic probabilistic aspects allows a design without large weight penalties.

Hammershock loading for dynamic response due to hammershock and the validation of the results is described in the first part of the paper. In the seconds part the deviation of the probabilistic approach is described for two different methods and the application on a current combat aircraft project including the refinement of the assumptions are discussed.

2.1 Hammershock loading

Aircraft with supersonic flight capability require an intake / duct in front of the engine because the engine cannot operate in supersonic flow conditions. Therefore the intake / duct have to be designed for subsonic flow conditions at the engine face (Fig. 1).

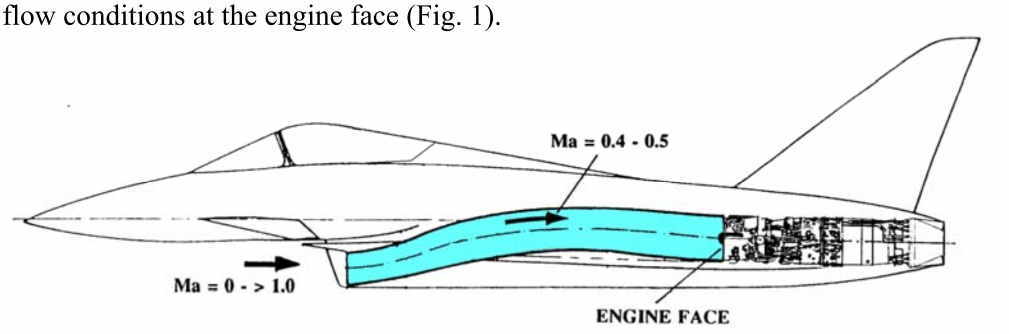

Fig. 1 Scheme of Intake / Duct of Supersonic Aircraft

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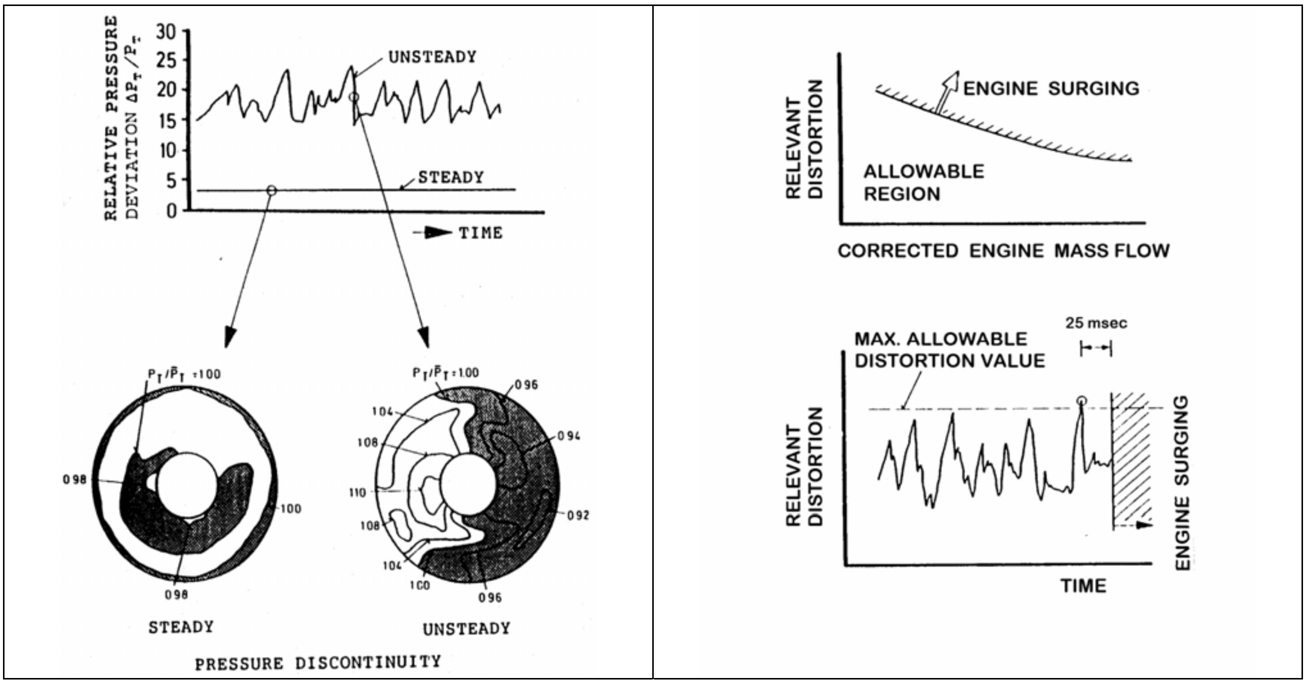


Fig. 2 Origin of Hammershock

Origin of hammershock (H.S.)
From the compressor region up to the combustion chamber a strong steady pressure increase occurs (Fig. 2). Near limit engine performance conditions a short-time pressure wave, called hammershock, can occur which advances in opposite direction of the airflow with high velocity (V_{HS} = 300 to 400 m/s). Under normal distortion free conditions there is a steady pressure increase of the air up to the combustion chamber of the engine. If a high unsteady pressure difference occurs which is caused by distortion of the air flow at limit engine performance conditions, an engine surge will occur, Fig. 2. This surge causes a very short pressure wave, which travels in opposite direction of the flow direction. The shock like wave - called hammershock (H.S.) - produces a pressure up to 3 times of the steady state pressure.

Effect of engine bypass ratio and compressor overall static pressure ratio on H.S. peak pressure
Increasing the compressor overall pressure ratio in general increases the ratio peak H.S. pressure to steady-state pressure at engine face and a decrease in engine bypass ratio leads to an increase in hammershock peak pressure.

Assumption of design H.S. pressure
The extrapolation of air intake H.S. peak pressures from existing engines has to be based upon the evaluation of the root mean square value added to the mean pressure as function of the overall static compressor pressure ratio. The peak H.S. pressure is then chosen as 3 times or 2 times of the root mean square value added to the mean value pending design assumptions.

Description of dynamic hammershock wave
The definition of the design hammershock wave is in general derived from experimental on ground surge - and flight test surge tests. Measurements performed on different aircraft at the engine face show typical time histories of the pressure at A.I.P. (air intake pressure), see for example Fig. 4. The general evaluation of a set of time histories will allow a definition of the H.S. pressure time history for subsonic and supersonic flight condition as demonstrated in Fig. 5. Important for dynamic response is, besides the magnitude of the peak value, the rise time to the positive peak value (values from 5 msec's down to 0.6 msec's have been measured) and the rise time to the negative peak value. It has to be noted that the negative H.S. pressure wave resulted from the reflected H.S. at the forward intake.

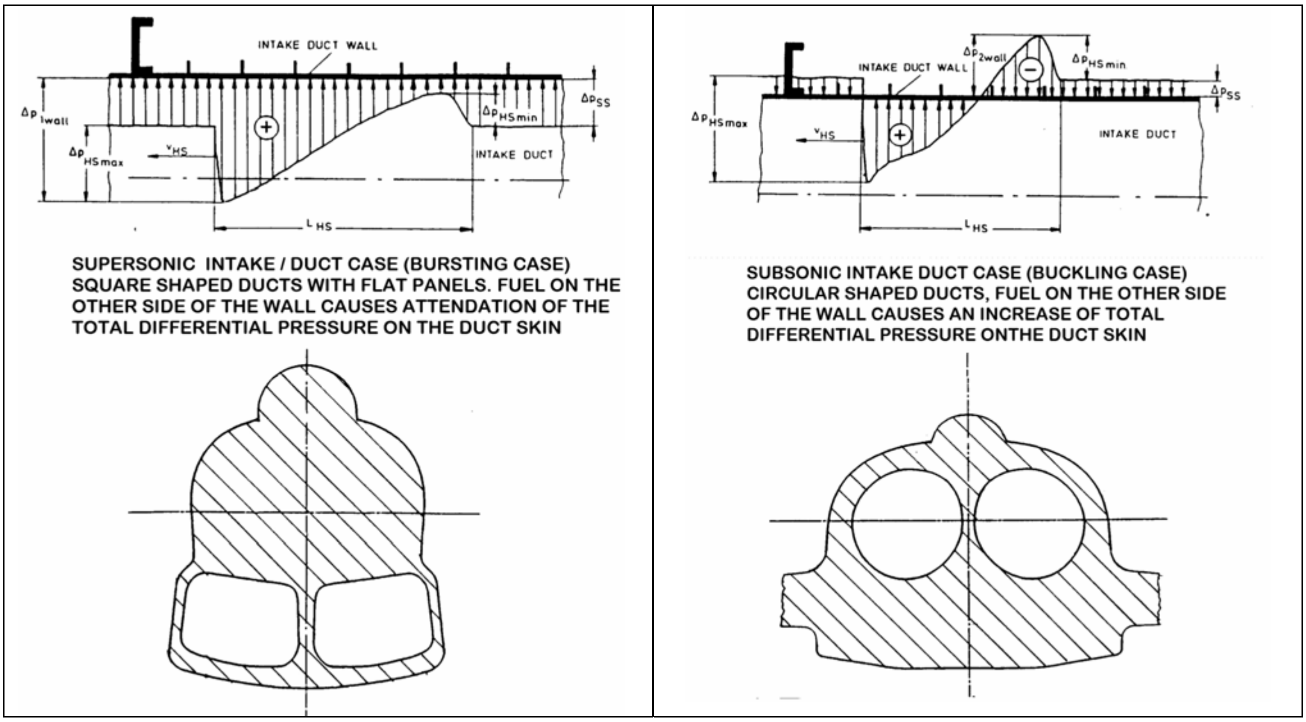
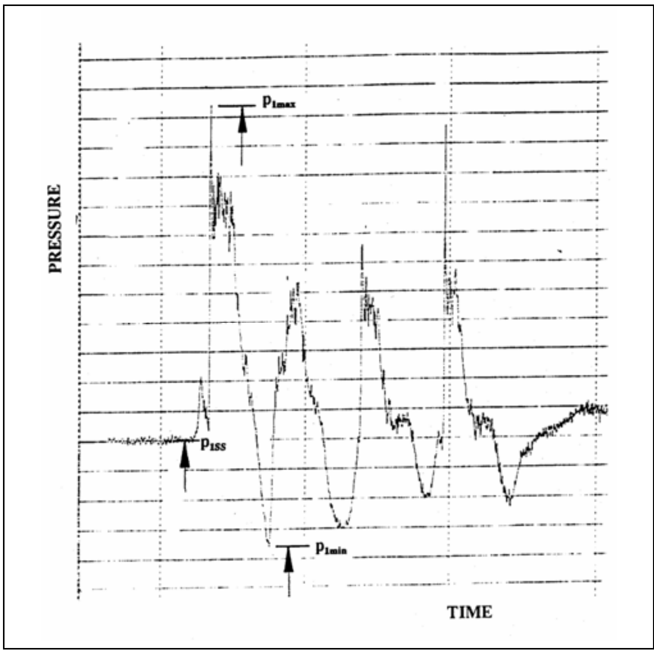


Fig. 3 Influence of Intake/Duct Shape on Total Pressure Including Hammershock



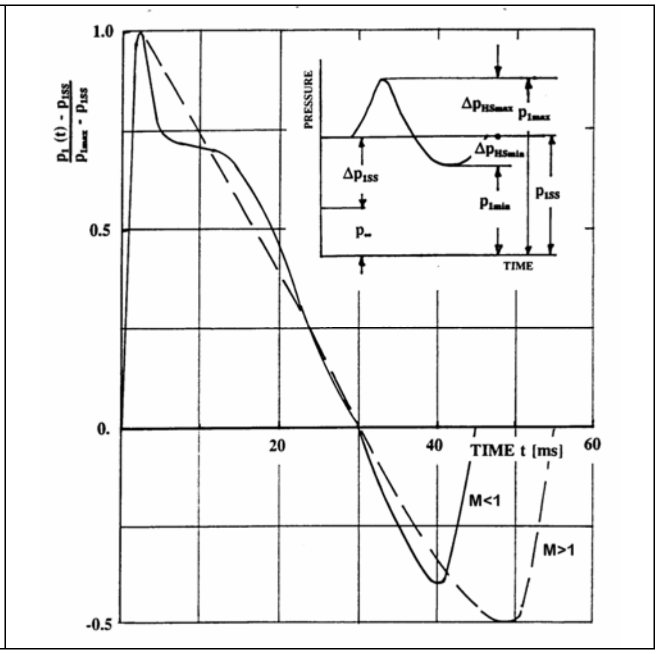


Fig. 4 In-flight measured Time History of Hammershock at Engine Face, Subsonic

Fig. 5 Hammershock Wave Shapes for Supersonic and Subsonic at Engine Face Derived from Experience

<u>Effect of duct cross section on design</u>
Different duct cross section shapes lead to different design conditions (Fig. 3)

- Supersonic intake duct case in the square shaped duct membrane stresses is critical for the more flat panels. In the tank region additional tank hydrostatic and tank system pressures (p_{TH} and p_{TV}) cause an attenuation of the total differential pressure on duct skin.

- Subsonic intake duct case in the round shaped duct stability requirements design the panels. In the tank region additional tank hydrostatic and tank system pressures (p_{TH} and p_{TV}) cause an increase of the total differential pressure on duct skin.

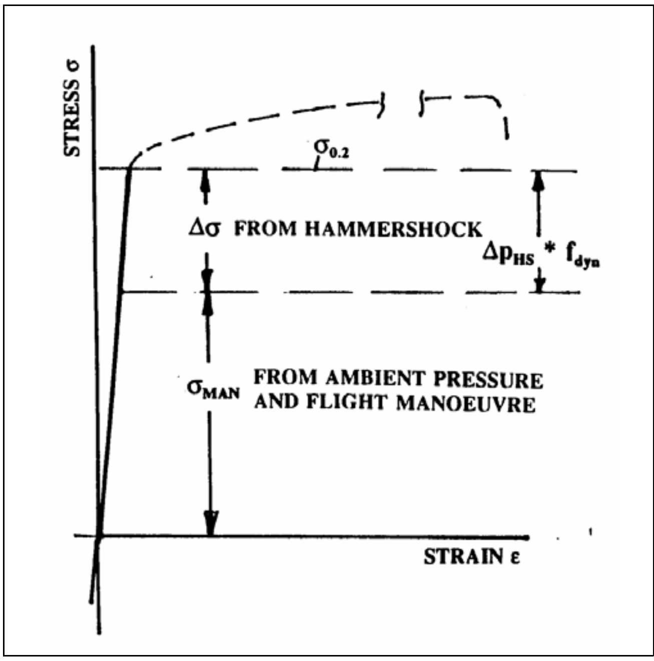


Fig. 6a Air Intake/Duct Design Stress/Strain Relation for Limit Load Case

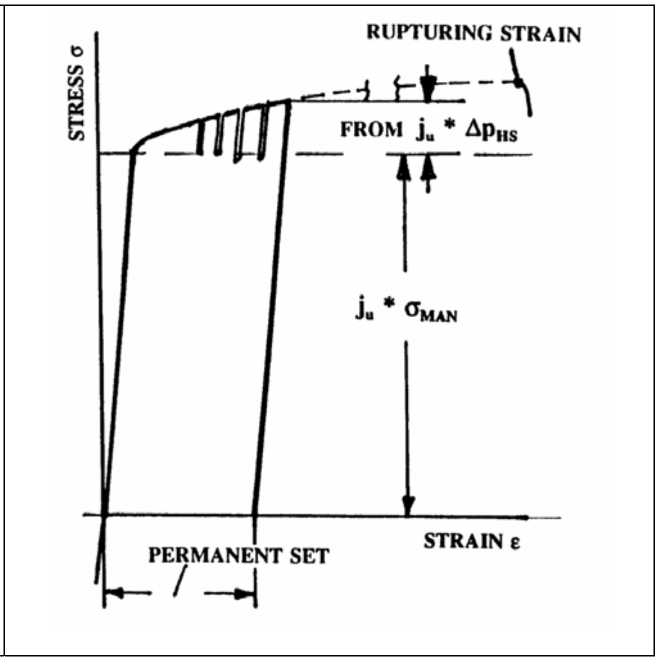


Fig. 6b Air Intake/Duct Design Stress/Strain Relation for Ultimate Load

2.2 Analytical procedure

The intake / duct structure has to be analysed in the different design steps in order to calculate the resulting stresses on the intake / duct panels and frames due to a total loading from manoeuvre 'g' loads, steady-state pressure, hydrostatic pressure from fuel and dynamic hammershock pressure.

In order to perform dynamic calculations a finite element model (FEM) has to be established which is able to describe local structural responses up to 5 kHz, i.e. to cover essential panel vibration modes and which has the capability to introduce the static loads or displacements from manoeuvres and steady-state pressures. In general for dynamic calculations an existing static FEM is modified by subdividing each of the original elements according to the frequency resolution requirement, see Fig. 7.

A full structural idealisation of the total intake / duct structure which would fulfil this requirement is not feasible at the moment due to the enormous complexity of the model leading to computer capacity and computer time problems which would hinder a practicable approach. Therefore different structural sections of the duct have to be treated.

<u>Calculation tools for limit load and ultimate load case</u>
Different tools are applied in the dynamic investigations. In the first step, the natural frequencies and elastic mode shapes are calculated using <u>NASTRAN SOLUTION 63</u> for model vibration analysis. In the second step for the investigation of the stresses and dynamic displacements in the limit load case NASTRAN SOLUTION 109 is applied for transient response analysis, with and without static preload using a dynamic hammershock load as described in chapter 2.1, Fig. 5. The properties of the geometry and of the elastic materials are linear.

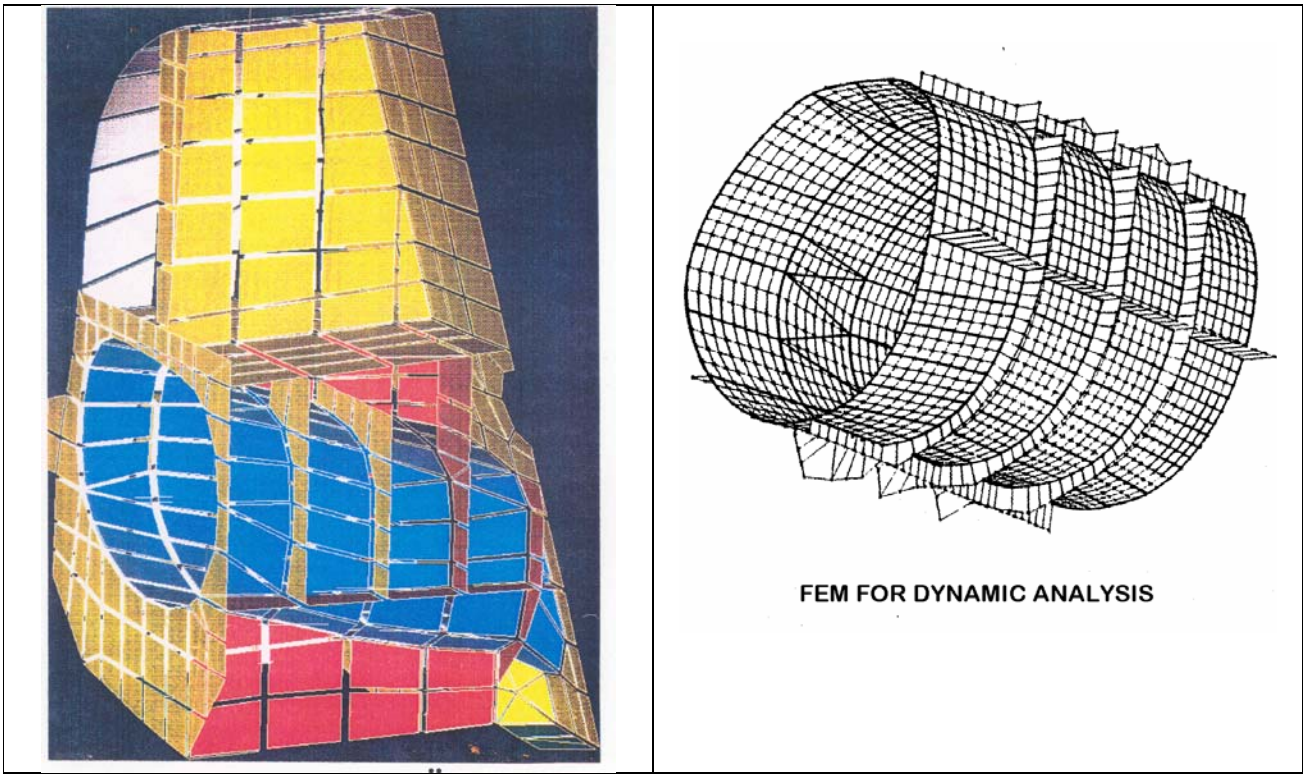


Fig. 7 Finite Element Models of Intake/Duct Structures

During the dynamic calculation local concentrations of high displacements and corresponding stress nests occur, which change in magnitude and position with the travel of the hammershock wave. The maximum stresses may remain for limit load case within the stress limit σ_{02} with the effect of the dynamic response covered by a dynamic factor on hammershock pressure in static design required the design philosophy. For this case the verification of the static design can be performed with NASTRAN SOLUTION 109, assuming that geometrical nonlinearities are not significant. For the ultimate load case the design philosophy might be based on the assumption that the duct structure is designed to ultimate loads from flight manoeuvres including steady state duct and hydrostatic pressures only, where the stress pulse from the ultimate hammershock pressure increment is covered by the plastic deformation capability of the duct material, see Fig. 6a+b. For this approach, a non-linear dynamic calculation with DYNA3D including non-linear plastic material description and non-linear geometrical properties is necessary for verification.

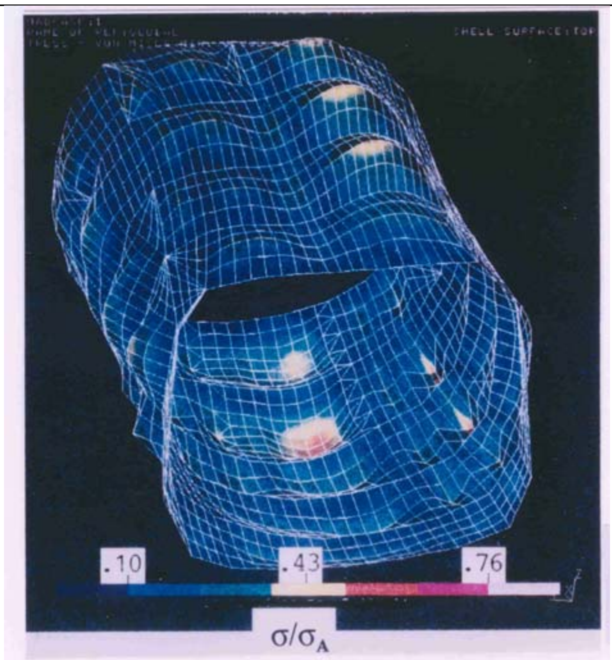
2.3 Results - Validation of Tools and Comparison of different methods

Dynamic hammershock calculations have been performed on supersonic squared shaped and subsonic circular shaped duct sections using NASTRAN SOL 109 and DYNA3D software in order to verify analytical tools and to verify dynamic factors used in the static design. Fig. 7 demonstrates that the original FEM for static calculations has to be refined for dynamic calculations from the frequency resolution point of view. A typical example of a dynamic model was a FEM consisting of 2348 grids, 12164 degree of freedom, 2073 QUAD elements, 785 triangular elements, 631 bar elements and 608 rod elements. The verification of an assumed dynamic factor in static design is demonstrated in the comparison of a static calculation with increased hammershock load and a dynamic calculation using SOLUTION 109 with hammershock acting from 0 to 10 ms on the structure in Fig. 8 and 9.

The comparison of static and dynamic calculation results resulted in an almost equivalent ratio of maximum stress to allowable stress $(\sigma/\sigma_A)_{static} = 0.57$ $(\sigma/\sigma_A)_{dyn} = 0.58$. Fig. 9 shows in addition that large structural portions have smaller stress levels than seen from the static calculation, Fig. 8.
The comparison of the different methods SOLUTION 109 with DYNA3D resulted in excellent agreement for limit load case investigations.

Comparison with experimental results

The dynamic response was measured in terms of strains on a duct test section. A finite element model of the test section was used for calculation of strains with the DYNA3D model using an input the experimental pressure pulse. The comparison of measured and calculated strain was reasonable good to validate the calculation.



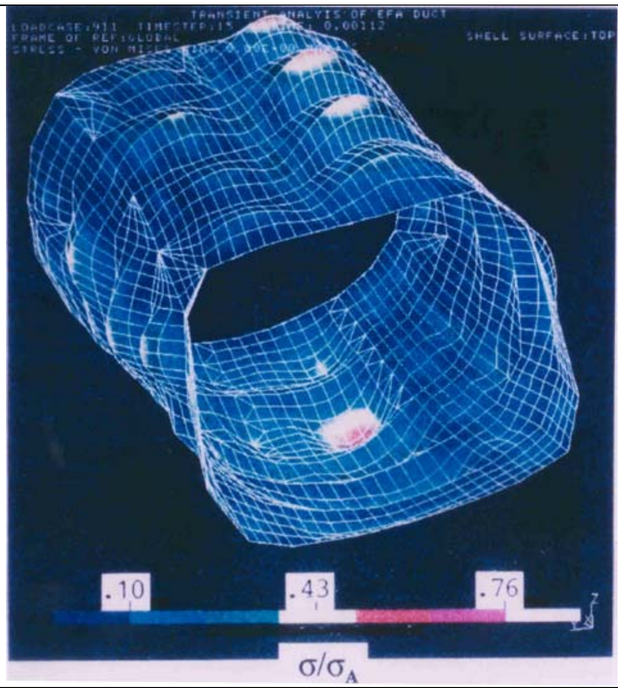


Fig. 8 Increased Static Hammershock Load Displacement and Stresses; Max Stress / Allowable Stress σ/σ_A =0.58.

Fig. 9 Increased Dynamic Hammershock Load Displacement and Stresses; Max Stress / Allowable Stress σ/σ_A =0.57.

2.4 Conclusions Hammershock Loads

- There is sufficient evidence for the application of software tools from the performed comparisons of calculated results using NASTRAN SOLUTION 109 and DYNA3D and from the comparison from calculated and measured dynamic strains for dynamic hammershock response and stress calculation in the process of the verification of air intake - duct structure.

- Comparison of local dynamic stress calculations to static stress calculations with assumed constant dynamic load factors (based on identical FEM) indicate that the dynamic tools could be applied not only for verification but also for the design to minimise structural weight.
  The dynamic design approach is relatively more complex and time consuming. The profits of local dynamic design might be reduced by manufacturing constraints.

- The verification of the assumed magnitude of the hammershock pressure and its risetime for a given shape of duct is the most important step for structural clearance.

3. Probability Analysis - Theory

This part of the report presents the results of a probability analysis to determine the number of occurrences of design hammershock (H.S.) pressures. An equation has been derived by methods of probability theory depending mainly on the distribution of the following random variables

- Intensity of H.S. ξ
- temperature T invested conditions: cold day, ISA day, hot day
- equivalent air-speed v

and on the following data

- frequency of H.S. (surge frequency 1.5/1000 engine hours)
- total design hammershock pressure vs. speed
- Aircraft fleet life (800 A/C's (typical feet size) times 6000 flight hours = 4.8 10^6 hours)

As an example with suitable assumptions of the distributions of the above mentioned variables and of the data have been made and the general result has been applied to three different total design H.S. pressures. To check the sensibility of the results a refinement of the assumptions of the above example has been made. In this approach the nonlinearity of the H.S. nature is tried to introduce into the analysis. The effects are shown below.

In addition a Monte Carlo simulation of the problem has been carried out. The results of the two different approaches have been compared and conclusions have been made.

3.1 The Need for a Probability Analysis in Designing for H.S. Loading

Generally speaking hammershock can be regarded as an unpredictable phenomenon randomly distributed throughout the flight envelope of jet aircraft.

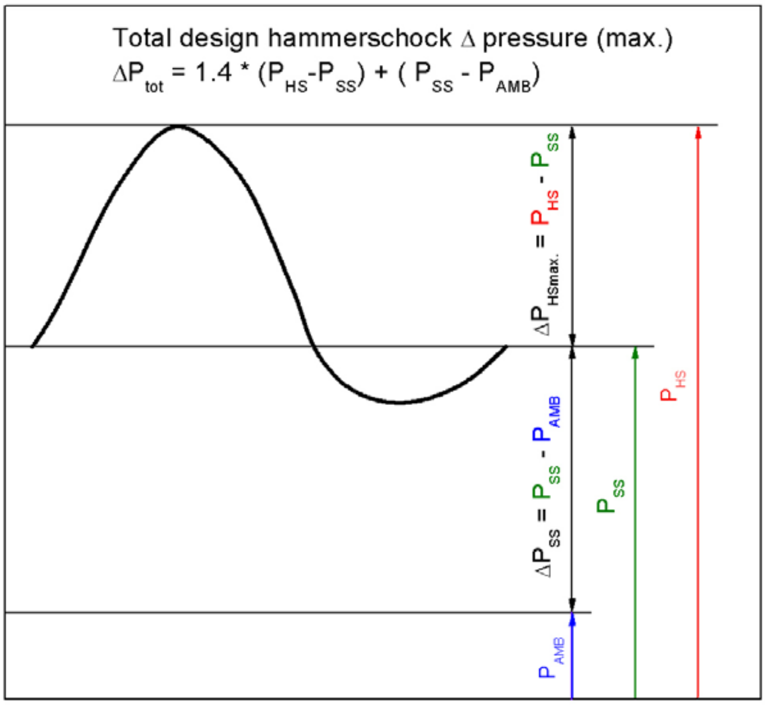


Fig. 10 Definitions of hammershock pressures and typical behavior versus time

Nevertheless it is desirable to know how many times a design H.S. pressure value is likely to be exceeded during the aircraft fleet life, i.e. it is important to know the distribution of the total design hammershock Δ pressure (max.) at AIP, see Fig. 10.

$$\Delta p_{tot} = 1.4 \cdot (p_{HS} - p_{SS}) + (p_{ss} - p_{AMB}),$$ where

p_{HS} = hammershock pressure
p_{SS} = steady state pressure and
p_{AMB} = ambient pressure

3.2 Mathematical model (Ref. 2-4)

The occurrence of a H.S. has been considered to be a stationary random process consisting of a sequence of impulses at the random times $T_i, i = 1,2\ldots$, of intensity Δp_{tot}, where T_i is exponential with parameter λ (= frequency of H.S.) and Δp_{tot} is the random variable total design hammershock pressure defined as in chapter 3.1, whose probability density function $f_{\Delta ptot}(p)$ will be derived in the following paragraph 3.3.

In symbols $X_t = \Delta p_{tot} \cdot \zeta_t$

where $\zeta_t = \sum_{i=1}^{\infty} \delta(t - T_i)$ and $\delta(t)$ is the well-known DIRAC distribution impulse 'function', see Fig. 11.

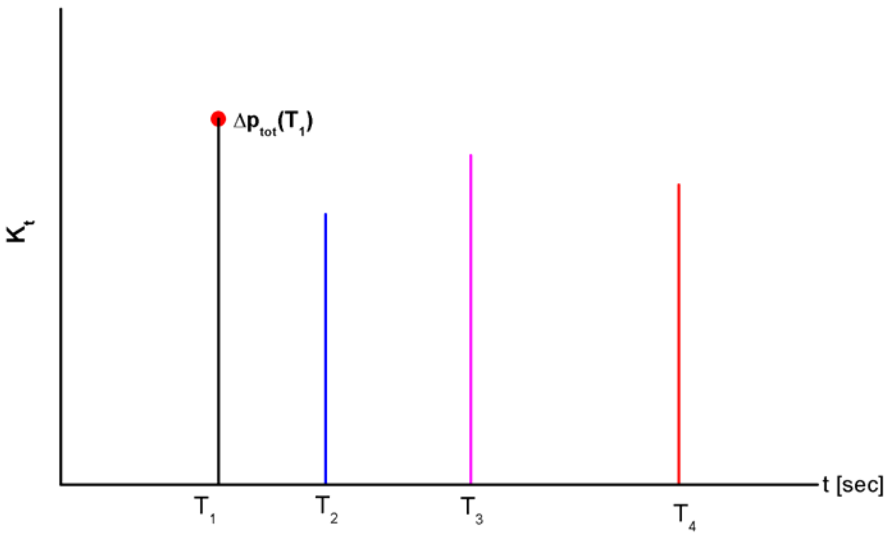


Fig. 11 Dirac impulse function used for probability analysis

The number of H.S. events $(N_t)_{t\geq 0}$ in an interval [0, t] of length t is a discrete-state process consisting of a family of increasing staircase functions with discontinuities at the points T_i (see Fig. 12).

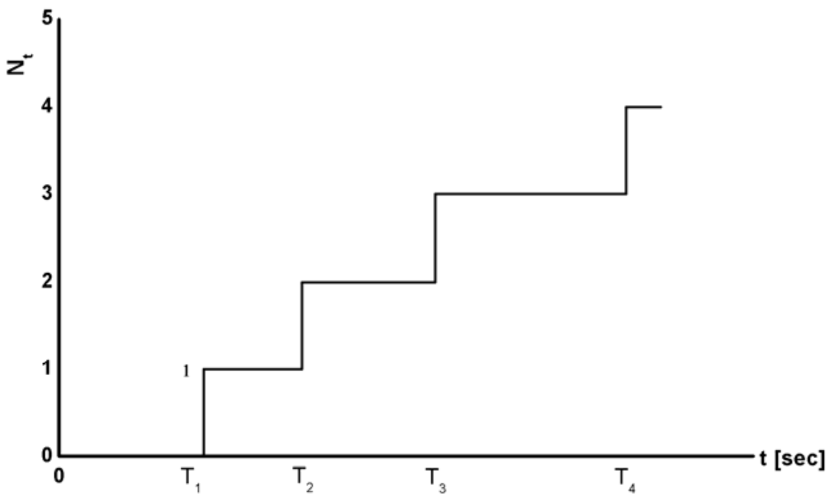


Fig. 12 Stair function

$(N_t)_{t\geq 0}$ is assumed to be a POISSON process, therefore for a specific t, N_t is a POISSON random variable with parameter $\lambda \cdot t$ (= mean number of events in a time interval [0, t])

Hence, for k = 0, 1, 2, ...

$$\text{prob } (N_t = k) = \frac{(\lambda \cdot t)^k}{k!} e^{-\lambda t}$$

And $IE(N_t) = \sum_{k=0}^{\infty} k \cdot prob(N_t = k) = \sum_{k=1}^{\infty} k \frac{(\lambda \cdot t)^k}{k!} e^{-\lambda t} = \lambda \cdot t$

Note that $\frac{d}{dt} N_t = \sum_{i=1}^{\infty} \delta(t - T_i) = \zeta_t$ (in distribution sense). Hence ζ_t gives the rate of occurrence of H.S.

In order to determine the expected rate of occurrence of H.S. pressure with values between a and b, a < b, the process

$$N_{a,b,t} = [\sigma(\Delta p_{tot} - a) - \sigma(\Delta p_{tot} - b)] \cdot \zeta_t$$

where $\sigma(x)$ is the unit step function, has been considered

The mean value of $N_{a,b,t}$ is equal to

$$IE(N_{a,b,t}) = IE(\sigma(\Delta p_{tot} - a) - \sigma(\Delta p_{tot} - b)) \cdot IE(\zeta_t),$$

since for every t and a, b the random variables $\sigma(\Delta p_{tot} - a) - \sigma(\Delta p_{tot} - b)$ and ζ_t are independent.

With these assumptions it follows

1) $IE(\sigma(\Delta p_{tot} - a)) = 1 \cdot prob(\sigma(\Delta p_{tot} - a) = 1) + 0 \cdot prob(\sigma(\Delta p_{tot} - b) = 0) = prob(\Delta p_{tot} > a)$

Similarly $IE(\sigma(\Delta p_{tot} - b)) = prob(\Delta p_{tot} > b)$
hence $IE(\sigma(\Delta p_{tot} - a)) - (\sigma(\Delta p_{tot} - b)) = prob(a < \Delta p_{tot} \leq b) = P_{a,b}$

2) $$IE(\zeta_t) = IE\left(\frac{d}{dt}N_t\right) = \frac{d}{dt}IE(N_t) = \frac{d}{dt}(\lambda \cdot t) = \lambda$$

Therefore $\lambda(a,b) := IE(N_{a,b,t}) = \lambda \cdot P_{a,b} = \lambda \cdot prob(a < \Delta p_{tot} \leq b)$ (1)

The rate of exceedance of the H.S. pressure is then given by $\lambda(a,\infty) = \lambda \cdot P_{a,\infty} = \lambda \cdot prob(\Delta p_{tot} > a)$ (2)

3.3 Derivation of the probability density of the random variable Δp_{tot}

In order to evaluate equation (1) and (2) it is necessary to determine the probability density function $f_{\Delta ptot}(p)$ of Δp_{tot}, since prob $(a < \Delta p_{tot} \leq b) = \int_a^b f_{\Delta ptot}(p) \cdot dp$

In this general part following assumptions have been made $\Delta p_{tot} = f(H, v, \xi, T)$ i.e. the design hammershock pressure is a function of the following random variables

- equivalent airspeed v_{EAS}
- H.S. intensity ξ
- temperature T
- altitude H

having, respectively the probability densities $f_v(x)$, $f_\xi(x)$, $f_T(x)$ and $f_H(x)$.

It should be pointed out, that

a) Instead of the equivalent airspeed one could use the random variable Ma-number.

b) This analysis has been restricted to the sea level, i.e. H = 0, so that $f_H(x)$ has not been used. In the general case the joint probability density $f_{v,H}(v,H)$ respectively $f_{Ma,H}(Ma,H)$ is required.

c) The dependence of Δp_{tot} on the temperature T can be expressed as follows:
For each day $T_i, i = 1,2,\ldots,n$, having a probability p_i, $p_1 + p_2 + \ldots + p_N = 1$, there is a function $\Delta p_{tot(i)} = f_i(H, v, \xi)$.

The derivation of $f_{\Delta ptot}(p)$ will follow in two steps. In the first one the probability density $f_{\Delta ptot(i)}(p)$ for each i = 1,2, ..., N will be derived. In the second step $f_{\Delta ptot}(p)$ will be solved.

1. Step
The following system with the auxiliary variables r = v, q = H has been considered (writing for simplicity p instead of $\Delta p_{tot(i)}$)

$$\begin{cases} p = f_i(H, v, \zeta) \\ q = H \\ r = v \end{cases} \tag{3}$$

The H.S. intensity ξ has been assumed to be independent of H and v.

Solving equation (3) with respect to ξ, H, v one obtains

$$\begin{cases} \xi = \phi_i(p, q, r) \\ H = q \\ v = r \end{cases} \tag{4}$$

Now, as ξ, H and v move about some probability volume in the (ξ, H, v)-space, the corresponding p, q, r move about the same probability volume in the (p, q, r)-space.

Hence

$$f_{p,q,r}(p,q,r) \cdot dp \cdot dq \cdot dr = f_{H,v,\xi}(H,v,\xi) \cdot dH \cdot dv \cdot d\xi \tag{5}$$

Because of the independence of ξ and (H, v) it follows

$$f_{p,q,r}(p,q,r) \cdot dp \cdot dq \cdot dr = f_{H,v}(H,v) \cdot f_{\xi}(\xi) \cdot dH \cdot dv \cdot d\xi \tag{6}$$

Now, the relation between elements of volume in the two spaces is determined by the well-known mathematical expression for the Jacobian transformation of coordinates, namely

$$dp \cdot dq \cdot dr = |J| dH \cdot dv \cdot d\xi \tag{7}$$

where $$J = \left| \frac{\partial(p,q,r)}{\partial(H,v,\xi)} \right| = \begin{vmatrix} \frac{\partial p}{\partial H} & \frac{\partial p}{\partial v} & \frac{\partial p}{\partial \xi} \\ \frac{\partial q}{\partial H} & \frac{\partial q}{\partial v} & \frac{\partial q}{\partial \xi} \\ \frac{\partial r}{\partial H} & \frac{\partial r}{\partial v} & \frac{\partial r}{\partial \xi} \end{vmatrix} \tag{8}$$

is the Jacobian determinant and the absolute value of J should be used since the probabilities $f_{p,q,r}(p,q,r)$ and $f_{H,v}(H,v) \cdot f_{\xi}(\xi)$ are positive quantities between zero and one.

Hence

$$f_{p,q,r}(q,p,r) = \frac{f_{H,v}(H,v) \cdot f_{\xi}(\xi)}{|J|} \tag{9}$$

Using this formula, the variables H, v, and ξ on the right-hand side must be replaced by their appropriate p, q, r.

Therefore

$$\frac{\partial p}{\partial H}=\frac{\partial f_i}{\partial H},\quad \frac{\partial p}{\partial v}=\frac{\partial f_i}{\partial v},\quad \frac{\partial p}{\partial \xi}=\frac{\partial f_i}{\partial \xi}$$
$$\frac{\partial q}{\partial H}=1,\quad \frac{\partial q}{\partial v}=0,\quad \frac{\partial q}{\partial \xi}=0 \quad ,$$
$$\frac{\partial r}{\partial H}=0,\quad \frac{\partial r}{\partial v}=1,\quad \frac{\partial r}{\partial \xi}=0$$

$$J=\begin{vmatrix} \frac{\partial f_i}{\partial H} & \frac{\partial f_i}{\partial v} & \frac{\partial f_i}{\partial \xi} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix}=-\frac{\partial f_i}{\partial \xi}$$

or $$|J|=\left|\frac{\partial f_i}{\partial \xi}(q,r,\xi)\right|,\quad \xi=\phi_i(p,r,q)$$

Finally

$$f_{p,q,r}(p,q,r)=\frac{f_{H,v}(q,r)\cdot f_{\xi}(\phi_i(q,r,p))}{\left|\frac{\partial f_i}{\partial \xi}(q,r,\xi)\right|} \tag{10}$$

This is the joint probability density function of $\Delta p_{tot(i)}$, H and v.

An integration over the altitude range results in the joint probability density function of $\Delta p_{tot(i)}$ and v.

$$f_{\Delta ptot(i),r}(p,r)=\int_0^{\infty}\frac{f_{H,v}(q,r)\cdot f_{\xi}(\phi_i(q,r,p))}{\left|\frac{\partial f_i}{\partial \xi}(q,r,\xi)\right|}\; dq \tag{11}$$

Integration over the speed range yields the probability density function of $\Delta p_{tot(i)}$ for the day i.

$$f_{\Delta ptot(i)}(p)=\int_0^{\infty} f_{p,r}(p,r)\; dr \tag{12}$$

2. Step

These functions combined with the theorem of the total probability, the joint probability density function of Δp_{tot} and v and the probability density function of Δp_{tot} can be derived. Indeed

$$\begin{aligned} F(p,v) &= prob\left(\Delta p_{tot}\le p,\ v_{EAS}\le v\right) \\ &= \sum_{i=1}^{N} prob\ (T=T_i)\cdot prob\left(\Delta p_{tot}\le p,\ v_{EAS}\le v\ \middle|T=T_i\right) \\ &= \sum_{i=1}^{N} p_i\cdot\int_{-\infty}^{p}\int_0^{v} f_{\Delta ptot(i),v}(s,t)\; ds\cdot dt \end{aligned}$$

Hence

$$f_{\Delta ptot,v}(p,v) = \frac{\partial^2 F}{\partial p \cdot \partial v} = \sum_{i=1}^{N} p_i \cdot f_{\Delta ptot(i),v}(p,v)$$ or using equation (11)

$$f_{\Delta ptot,v}(p,v) = \int_0^\infty \sum_{i=1}^{N} p_i \frac{f_{H,v}(q,v) \cdot f_\xi(\phi_i(q,v,p))}{\left| \frac{\partial f_i}{\partial \xi}(q,v,\xi) \right|} \, dq \tag{13}$$

Hence

$$\begin{aligned} f_{\Delta ptot}(p) &= \int_0^\infty f_{\Delta ptot,v}(p,v) \, dv \\ &= \int_0^\infty \int_0^\infty \sum_{i=1}^{N} p_i \frac{f_{H,v}(q,r) \cdot f_\xi(\phi_i(q,v,p))}{\left| \frac{\partial f_i}{\partial \xi}(q,v,\xi) \right|} \, dq \cdot dv \end{aligned} \tag{14}$$

The probability that Δp_{tot} falls in the range [a, b] and the speed v_{EAS} falls in the range $[v_a, v_e]$ is then given by

$$\begin{aligned} P_{a,b,v_a,v_e} &= prob\left(a \le \Delta p_{tot} \le b, \; v_a \le v_{EAS} \le v_e \right) \\ &= \int_a^b \int_{v_a}^{v_e} f_{\Delta ptot,v}(p,v) \cdot dp \cdot dv \end{aligned} \tag{15}$$

whereas the probability that Δp_{tot} falls in the range [a, b] is given by

$$P_{a,b} = prob\left(a \le \Delta p_{tot} \le b \right) = \int_a^b f_{\Delta ptot}(p) \cdot dp \tag{16}$$

To summarize $\lambda(a,b) = \lambda \cdot P_{a,b} = \lambda \cdot \int_a^b f_{\Delta ptot}(p) \cdot dp$ (17)

$f_{\Delta ptot}(p)$ according to equation (14), gives the rate of occurrence of the event $A = \{a \le \Delta p_{tot} \le b\}$. The rate of exceedance of a pressure value a is given by letting $b \to \infty$.

On the other hand, if it is required the rate of occurrence of the event $B = \{a \le \Delta p_{tot} \le b, v_a \le v_{EAS} \le v_b\}$, then

$$\lambda(a,b,v_a,v_e) = \lambda \cdot P_{a,b,v_a,v_e} = \lambda \cdot \int_a^b \int_{v_a}^{v_e} f_{\Delta ptot,v}(p,v) \cdot dp \cdot dv \tag{18}$$

and the rate of exceedance of the event B is given by letting $b \to \infty$.

The number of occurrences (respectively exceedances) of an event is given by multiplying the A/C fleet life TF by the rate of occurrence (respectively exceedance) of the event.

This completes the theoretical investigation for the development of proper formulae for hammershock exceedances.

A further simplification of the formulae (13) and (14) is possible if suitable assumptions of the functions $f_i, f_{H,v}, f_\xi$ are made. This will be shown on an example in the next chapter.

4. Applications

The above theory has been applied to determine the rate of exceedances and the number of exceedances per A/C fleet life of three different total H.S. pressures for a modern fighter aircraft. Following assumptions to evaluate formulae (17) and (18) have been made.

4.1 Assumptions

Temperature T

Discrete distribution with a probability function (n = 3)

i	day	p_i = Flying time at given Temperature conditions in [%]
1	COLD	15
2	ISA	50
3	HOT	35

(19)

Note that the term **COLD DAY** comprises COLD DAY with 5% and ISA DAY-15°C with 10% and
HOT DAY comprises HOT DAY with 5% and ISA DAY+15°C with 30%

Frequency of H.S.

$$\lambda = \frac{1.5}{1000 Engine \quad hrs} = \frac{3}{1000 \cdot A/C \quad hrs}$$

Aircraft fleet life

TF = 800 A/C * 6000 hrs = 4.8 10^6 hrs

Total design H.S. pressure vs. speed and altitude

Figs. 13 to 16 have been considered however only at sea level (this is in our opinion a pessimistic consideration) and in a linearized form, i.e. following linear approximation have been made.

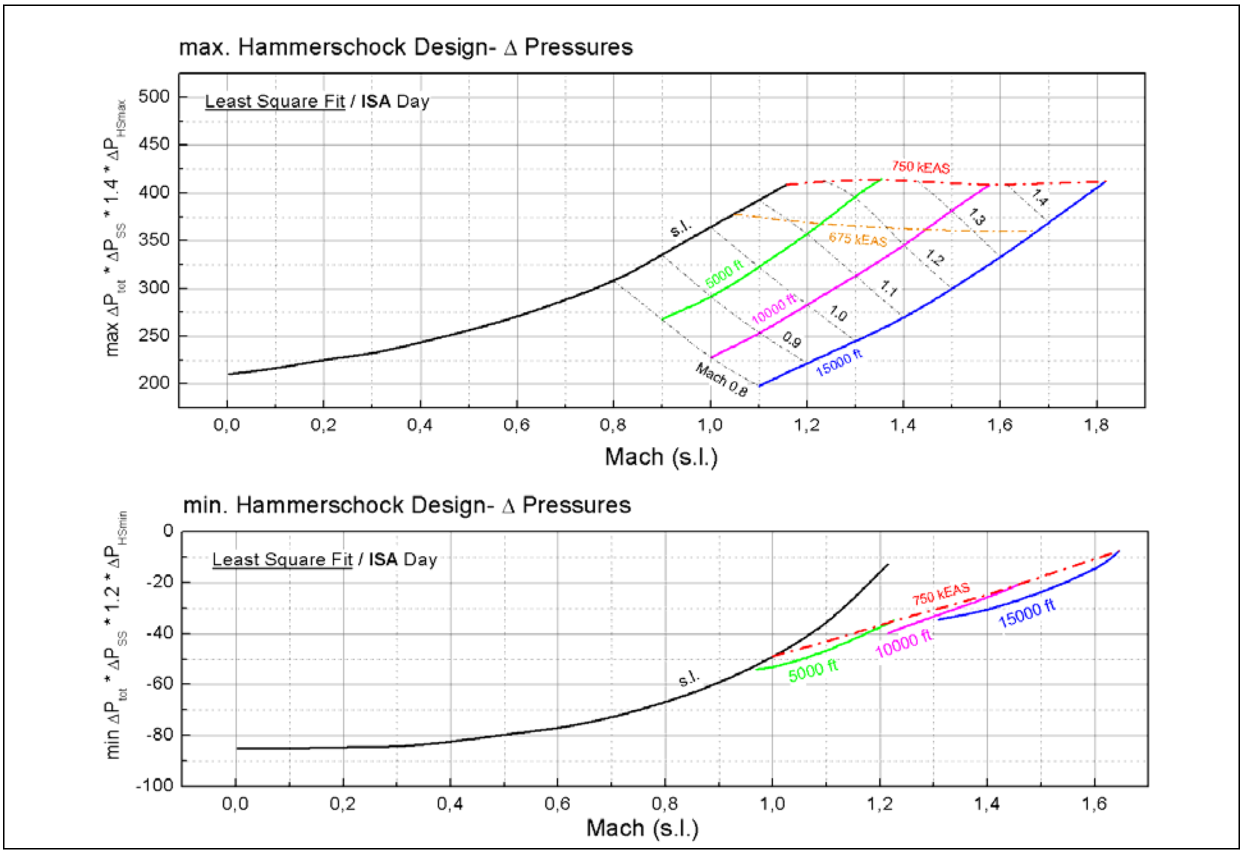


Fig. 13 Hammershock design pressure – ISA DAY - Least Square Fit

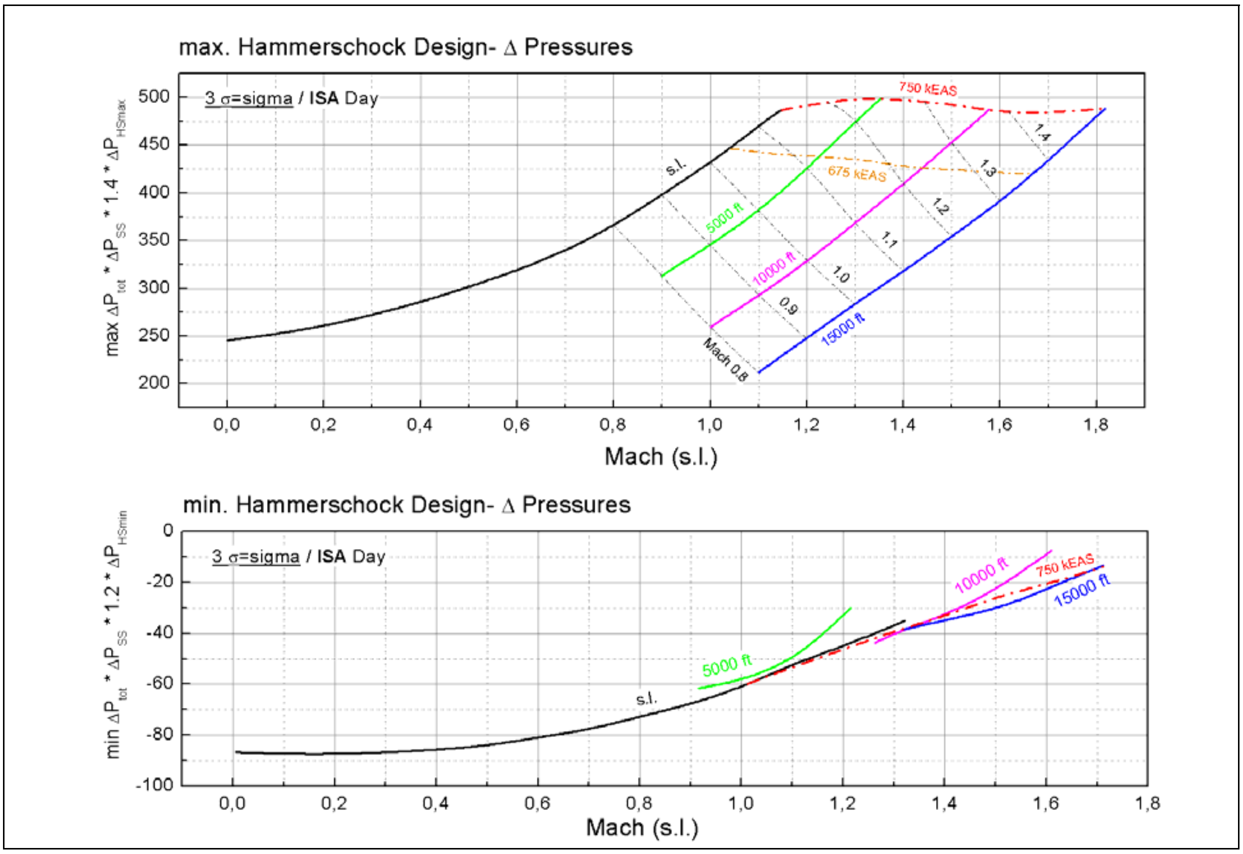


Fig. 14 Hammershock design pressure – ISA DAY – 3 Sigma

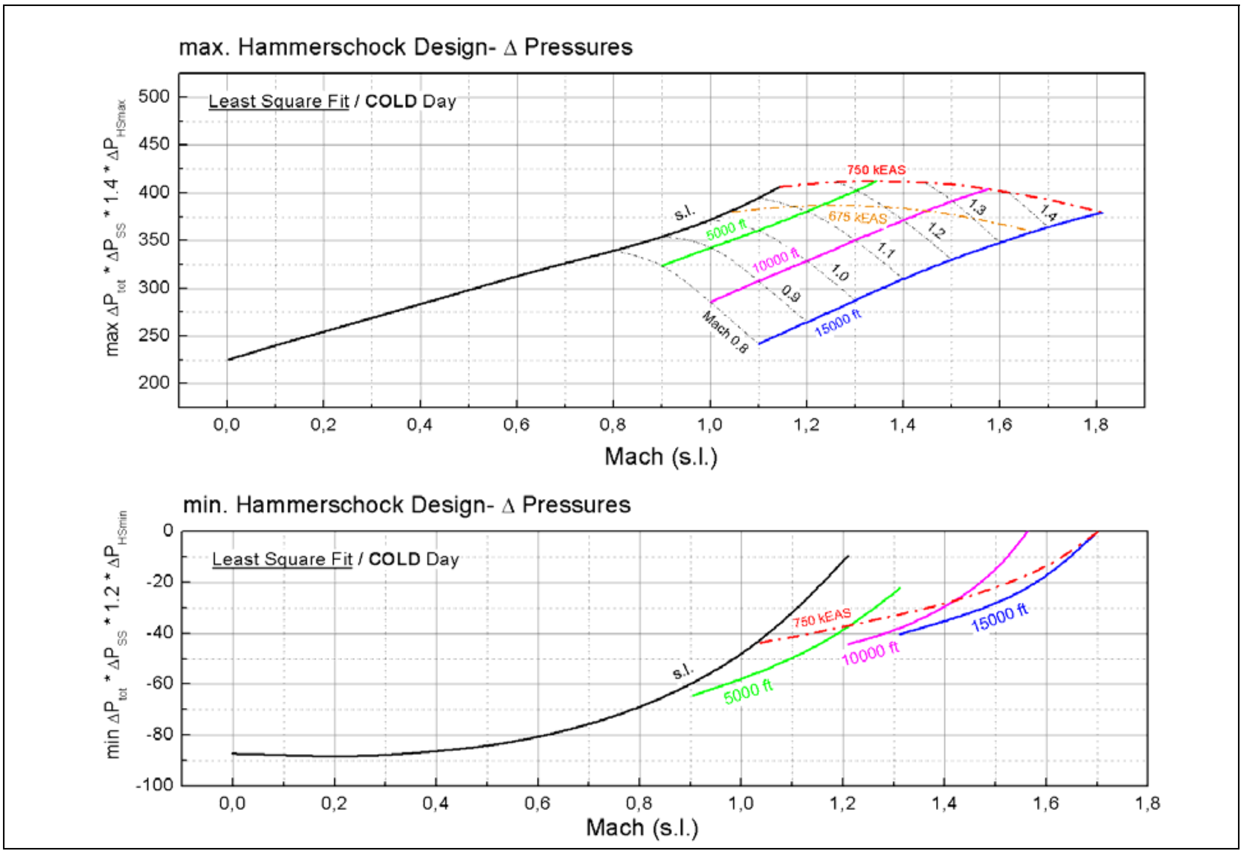


Fig. 15 Hammershock design pressure – COLD DAY - Least Square Fit

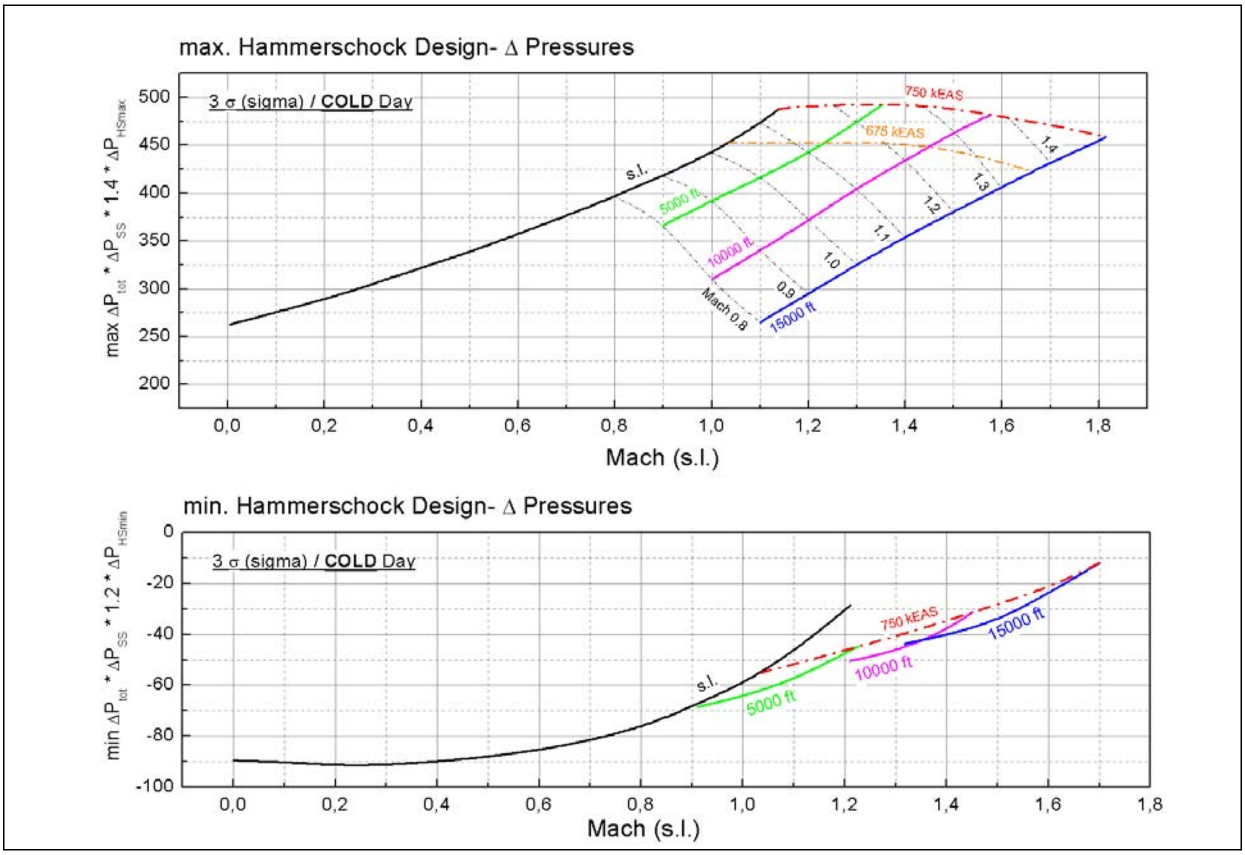


Fig. 16 Hammershock design pressure – COLD DAY – 3 Sigma

Standard Deviation \ Day	Cold Day	ISA Day
Least square fit	$f_{01}(v)=\alpha_{01}-\beta_{01}(v_e-v)$	$f_{02}(v)=\alpha_{02}-\beta_{02}(v_e-v)$
$3-\sigma$	$f_{11}(v)=\alpha_{11}-\beta_{11}(v_e-v)$	$f_{12}(v)=\alpha_{12}-\beta_{12}(v_e-v)$

(20)

where v_e = 750 [kts] EAS and

$\alpha_{01}=407.\quad[kPa]\qquad\beta_{01}=0.28\quad[kPa/Kts]$

$\alpha_{02}=405.\quad[kPa]\qquad\beta_{02}=0.37\quad[kPa/Kts]$

$\alpha_{11}=488.\quad[kPa]\qquad\beta_{11}=0.37\quad[kPa/Kts]$

$\alpha_{12}=482.\quad[kPa]\qquad\beta_{12}=0.45\quad[kPa/Kts]$

For hot day the following extrapolation has been made:

Standard Deviation \ Day	Hot Day
Least square fit	$f_{03}(v)=f_{02}(v)-(f_{01}(v)-f_{02}(v))=2\cdot f_{02}(v)-f_{01}(v)$
$3-\sigma$	$f_{13}(v)=f_{12}(v)-(f_{11}(v)-f_{12}(v))=2\cdot f_{12}(v)-f_{11}(v)$

(21)

or $f_{03}(v)=\alpha_{03}-\beta_{03}(v_e-v)$, $\qquad f_{13}(v)=\alpha_{13}-\beta_{13}(v_e-v)$,

where

$\alpha_{03}=403.\quad[kPa]\qquad\beta_{03}=0.46\quad[kPa/Kts]$

$\alpha_{13}=476.\quad[kPa]\qquad\beta_{13}=0.53\quad[kPa/Kts]$

see fig. 17.

By linear interpolation over the H.S. intensity ξ the $\Delta p_{tot(i)}$ function for an arbitrary multiple of the standard deviation of ξ has been derived (i=1, 2, 3),

$$\Delta p_{tot(i)}(H=0.,v,\xi)=f_{0i}(v)+\frac{\xi}{3}\left(f_{1i}(v)-f_{0i}(v)\right) \qquad (22)$$

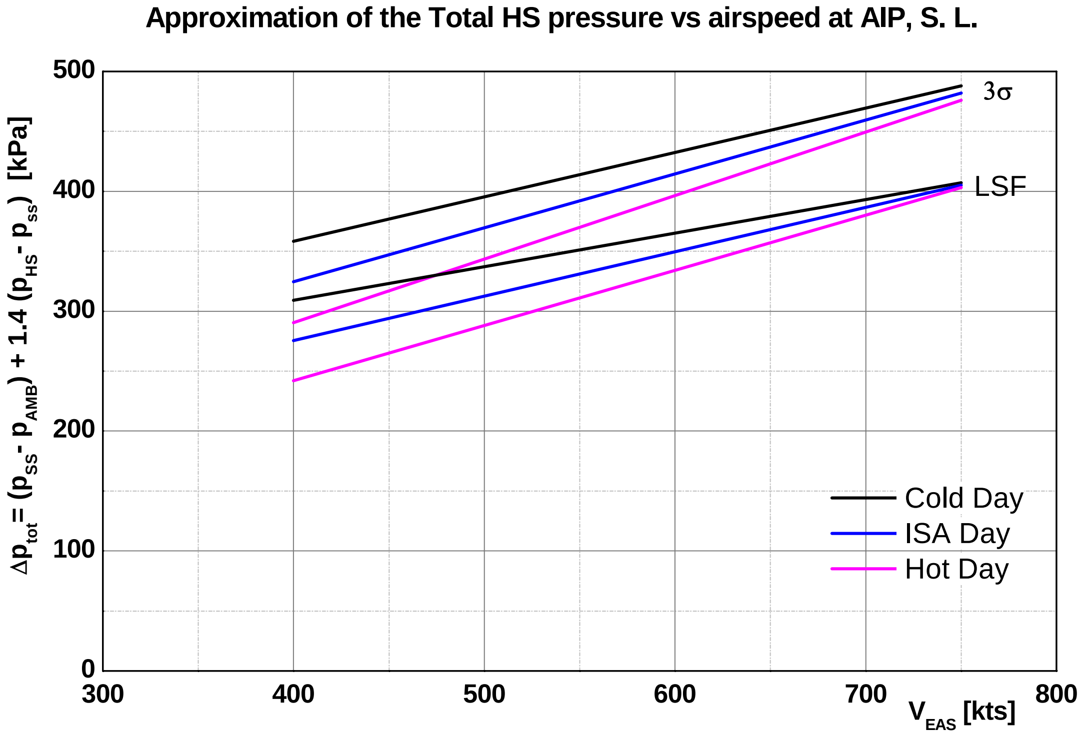


Fig. 17 Approximation of total Hammershock pressure vs. airspeed

Inserting Equation (1) and (2) into Equation (3) and after some simple algebraic transformations the following expressions (i=1, 2, 3) will be arrived.

$$\Delta p_{tot(i)}(H=0.,v,\xi)=\alpha_i+\beta_i v+\xi\cdot(\gamma_i+\delta_i\cdot v), \tag{23}$$

where

$\alpha_1=\alpha_{01}-\beta_{01}\cdot v_e=197. \quad [kPa]$

$\alpha_2=\alpha_{02}-\beta_{02}\cdot v_e=127.5 \quad [kPa]$

$\alpha_3=2(\alpha_{02}-\beta_{02.}\cdot v_e)-\alpha_{01}+\beta_{01}\cdot v_e=58. \quad [kPa]$

$\gamma_1=(\alpha_{11}-\alpha_{01}+v_e\cdot(\beta_{01}-\beta_{11}))/3=\dfrac{13.5}{3} \quad [kPa]$

$\gamma_2=(\alpha_{12}-\alpha_{02}+v_e\cdot(\beta_{02}-\beta_{12}))/3=\dfrac{17}{3} \quad [kPa]$

$\gamma_3=(2\cdot\gamma_2-\gamma_1)/3=\dfrac{20.5}{3} \quad [kPa]$

$\beta_1=\beta_{01}=0.28 \quad [kPa/Kts]$

$\beta_2=\beta_{02}=0.37 \quad [kPa/Kts]$

$\beta_3=2\cdot\beta_{02}-\beta_{01}=0.46 \quad [kPa/Kts]$

$\delta_1=(\beta_{11}-\beta_{01})/3=\dfrac{0.09}{3} \quad [kPa/Kts]$

$\delta_2=(\beta_{12}-\beta_{02})/3=\dfrac{0.08}{3} \quad [kPa/Kts]$

$\delta_3=(2\delta_2-\delta_1)/3=\dfrac{0.07}{3} \quad [kPa/Kts]$

Formula (23) gives the unknown functions $f_i(H=0.,v,\xi)$, it follows

$$\frac{\partial f_i}{\partial \xi}(q,r,\xi)=\gamma_i+\delta_i\cdot r, \quad \text{and} \quad \xi=\phi_i(q,r,p)=\frac{p-\alpha_i-\beta_i\cdot r}{\gamma_i+\delta_i\cdot r} \tag{24}$$

Now equations (13) and (14) can be simplified as follows

$$f_{\Delta ptot,v}(p,v)=\sum_{i=1}^{3} p_i \frac{f_v(v)\cdot f_\xi \cdot \left(\dfrac{p-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v}\right)}{\left|\gamma_i+\delta_i\cdot v\right|} \tag{25}$$

Note that there is no dependence on Altitude H and

$$f_{\Delta ptot}(p)=\int_0^\infty \sum_{i=1}^{3} p_i \frac{f_v(v)\cdot f_\xi \cdot \left(\dfrac{p-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v}\right)}{\left|\gamma_i+\delta_i\cdot v\right|}\cdot dv \tag{26}$$

With the aid of the transformation

$$\tau=\frac{p-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v} \qquad \text{and} \qquad d\tau=\frac{dp}{\gamma_i+\delta_i\cdot v}$$

$$a\le p\le b\rightarrow \tau_0:=\frac{a-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v}\le\tau\le\tau_1:=\frac{b-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v}$$

and using $\psi(\tau):=\int_{-\infty}^{\tau} f_\xi(v)\cdot dv$ formulae (15) and (16) are now

$$P_{a,b,v_a,v_e}=\int_{v_a}^{v_e} f_v(v)\cdot\left(\sum_{i=1}^{3} p_i\cdot sign(\gamma_i+\delta_i\cdot v)\cdot\left(\psi\left(\frac{b-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v}\right)-\psi\left(\frac{a-\alpha_i-\beta_i\cdot v}{\gamma_i+\delta_i\cdot v}\right)\right)\right)\cdot dv \tag{27}$$

and $P_{a,b}=P_{a,b,0,\infty}$ (28)

where $sign(x):=\begin{cases} +1 & for \quad x>0 \\ 0 & for \quad x=0 \\ -1 & for \quad x<0 \end{cases}$

is the well known sign function.

Hammershock intensity ξ

Normal (Gaussian) distribution with a probability density is given by:

$$f_\xi(x):=\frac{1}{\sqrt{2\pi}}\cdot e^{-\frac{1}{2}x^2}$$

Mean value $\mu=0$ and the standard deviation $\sigma=1$

This choice of $f_\xi(x)$ leads to a further simplification of equations (26) - (28).

Since

$$\psi(x):=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\cdot dt=\frac{1}{2}\left(1+erf\left(\frac{x}{\sqrt{2}}\right)\right) \quad \text{where}$$

$erf(x):=\dfrac{2}{\sqrt{\pi}}\cdot\int_0^x e^{-t^2}\cdot dt$ is the error function and with $sign(\gamma_i+\delta_i\cdot v)=+1 \quad for \quad i=1,2,3$

it follows

$$P_{a,b,v_a,v_e} = \int_{v_a}^{v_e} f_v(v) \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \cdot \left(erf\left(\frac{b-\alpha_i-\beta_i \cdot v}{\sqrt{2}(\gamma_i+\delta_i \cdot v)} \right) - erf\left(\frac{a-\alpha_i-\beta_i \cdot v}{\sqrt{2}(\gamma_i+\delta_i \cdot v)} \right) \right) \right) \cdot dv \qquad (29)$$

and

$$P_{a,b} = \int_{0}^{\infty} f_v(v) \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \cdot \left(erf\left(\frac{b-\alpha_i-\beta_i \cdot v}{\sqrt{2}(\gamma_i+\delta_i \cdot v)} \right) - erf\left(\frac{a-\alpha_i-\beta_i \cdot v}{\sqrt{2}(\gamma_i+\delta_i \cdot v)} \right) \right) \right) \cdot dv \qquad (30)$$

The probability of the events $\{\Delta p_{tot} > a, v_a \le v_{EAS} \le v_e\}$ and $\{\Delta p_{tot} > a\}$ are now

$$P_{a,\infty,v_a,v_e} = \int_{v_a}^{v_e} f_v(v) \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \cdot \left(erfc\left(\frac{a-\alpha_i-\beta_i \cdot v}{\sqrt{2}(\gamma_i+\delta_i \cdot v)} \right) \right) \right) \cdot dv \qquad (31)$$

and $P_{a,\infty} = \int_{0}^{\infty} f_v(v) \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \cdot \left(erfc\left(\frac{a-\alpha_i-\beta_i \cdot v}{\sqrt{2}(\gamma_i+\delta_i \cdot v)} \right) \right) \right) \cdot dv$ (32)

since $erf(\infty) = 1$ *and* $erfc(x) := 1 - erf(x)$ is the complementary error function.

Speed Spectrum v(t)

Five different speed spectra have been taken into account: (Table 1)

Speed [Kts]	mins pro hour spent in a given speed range				
	C mission	E mission	G mission	F mission	W mission
0 < v < 50	0.1	0.15	0.03	0.1	0.15
50 < v < 100	0.2	0.2	0.13	0.2	0.2
100 < v < 150	0.3	0.3	0.18	0.6	0.3
150 < v < 200	3.3	1.0	3.64	1.4	1.5
200 < v < 250	5.0	1.5	1.85	3.5	3.5
250 < v < 300	8.9	2.2	19.16	10.0	4.5
300 < v < 350	1.6	2.7	2.462	25.0	6.35
350 < v < 400	17.1	3.3	22.3	7.8	3.15
400 < v < 450	12.8	30.0	6.47	6.2	20.0
450 < v < 500	3.5	10.0	2.88	3.8	10.0
500 < v < 550	6.7	5.0	0.331	1.0	6.7
550 < v < 600	0.3	2.0	0.358	0.3	2.0
600 < v < 650	0.2	1.0	0.084	0.1	1.0
650 < v < 700	0.0	0.5	0.125	0.0	0.5
700 < v < 750	0.0	0.15	0.0	0.0	0.15

Table 1 Equivalent Airspeed spectra

The spectrum of the F mission is shown graphically in Fig. 18

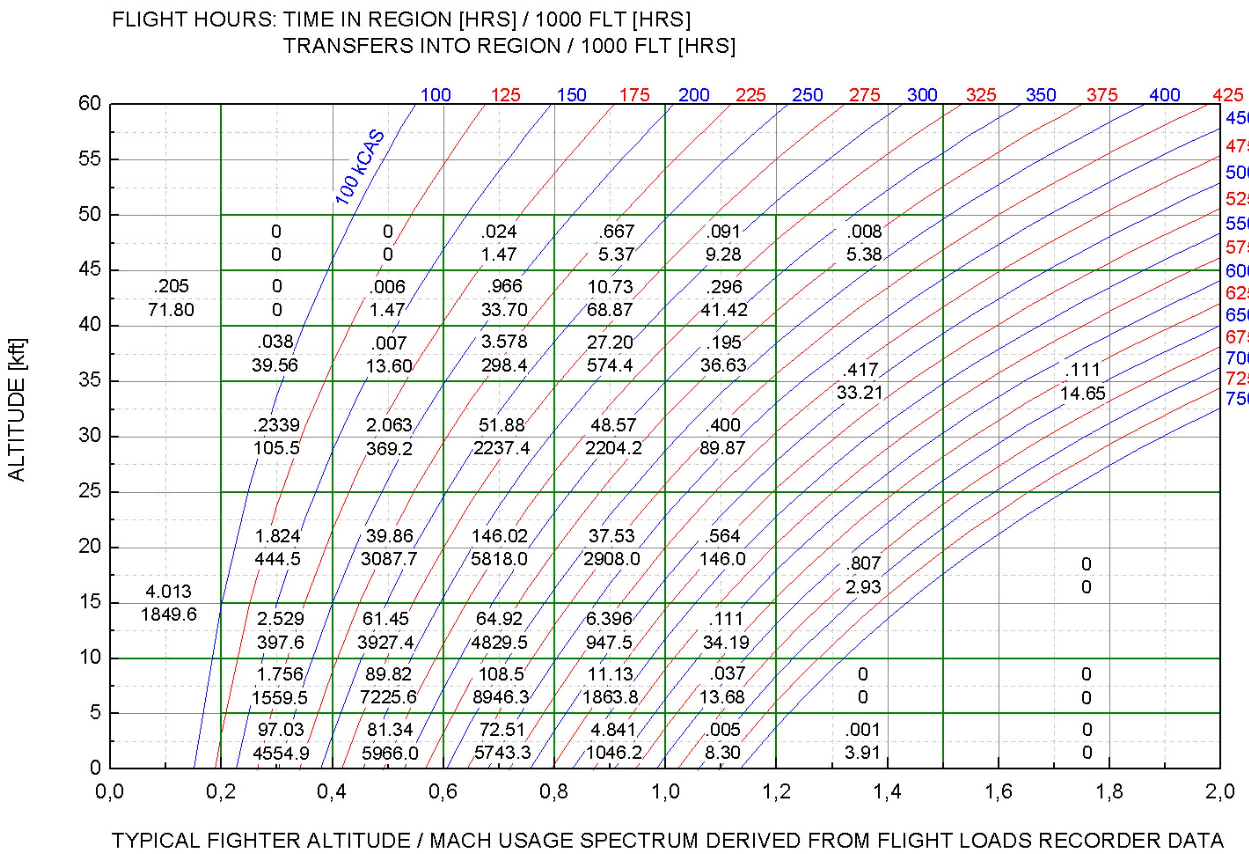


Fig. 18 Typical Fighter mission spectrum

The probability density function of the equivalent airspeed v_{EAS} is then given by

$$f_v(t) = \frac{v(t)}{3000.}$$

On account of completeness the mean value $\mu = \int_0^\infty t \cdot f_v(t) \cdot dt$ and the standard deviation

$\sigma = \int_0^\infty (t-\mu)^2 f_v(t) \cdot dt$ of the equivalent airspeed v_{EAS} are given:

Speed spectrum	μ [Kts]	σ [Kts]
C mission	367.58	102.851
E mission	427.25	91.259
G mission	335.87	83.214
F mission	335.42	79.345
W mission	409.63	108.162

(33)

A computer programme has been developed to calculate by numerical integration the formulas 26, 29, 30, 31 and 32.

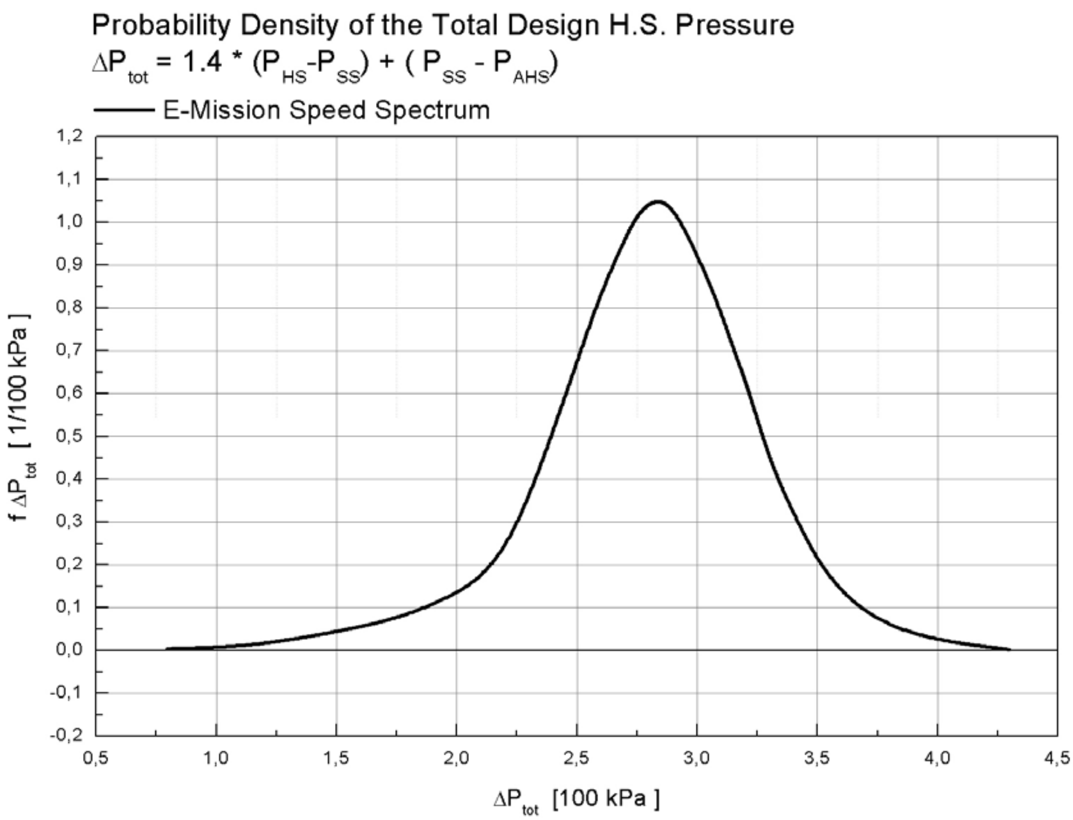


Fig. 19 Probability density of the total design

Total H.S. pressure at AIP [kPa]	Rate of occurence of H.S. pressure in given range per flying hour [1 / FH]				
	C mission	E mission	G mission	F mission	W mission
$300 \le p < 325$	3.650E-04	5.500E-04	2.083E-04	2.060E-04	5.000E-04
$325 \le p < 350$	1.650E-04	2.650E-04	6.100E-05	6.650E-05	2.640E-04
$350 \le p < 375$	4.440E-05	9.280E-05	1.455E-05	1.590E-05	9.820E-05
$375 \le p < 400$	7.010E-06	2.995E-05	4.373E-06	2.680E-06	3.100E-05
$400 \le p < 425$	6.430E-07	8.215E-06	1.128E-06	3.000E-07	8.260E-06
$425 \le p < 450$	3.590E-08	1.524E-06	1.602E-07	1.870E-08	1.525E-06
$450 \le p < 475$	1.070E-09	1.620E-07	1.070E-08	5.540E-10	1.620E-07
$475 \le p < 500$	1.400E-11	8.924E-09	3.220E-10	7.100E-12	8.924E-09
$500 \le p < 525$	7.320E-14	2.413E-10	4.300E-12	3.700E-14	2.413E-12
$525 \le p < 550$	1.440E-16	3.140E-12	2.500E-14	7.200E-17	3.140E-12

Table 2 Rate of occurrence of Hammershock pressure

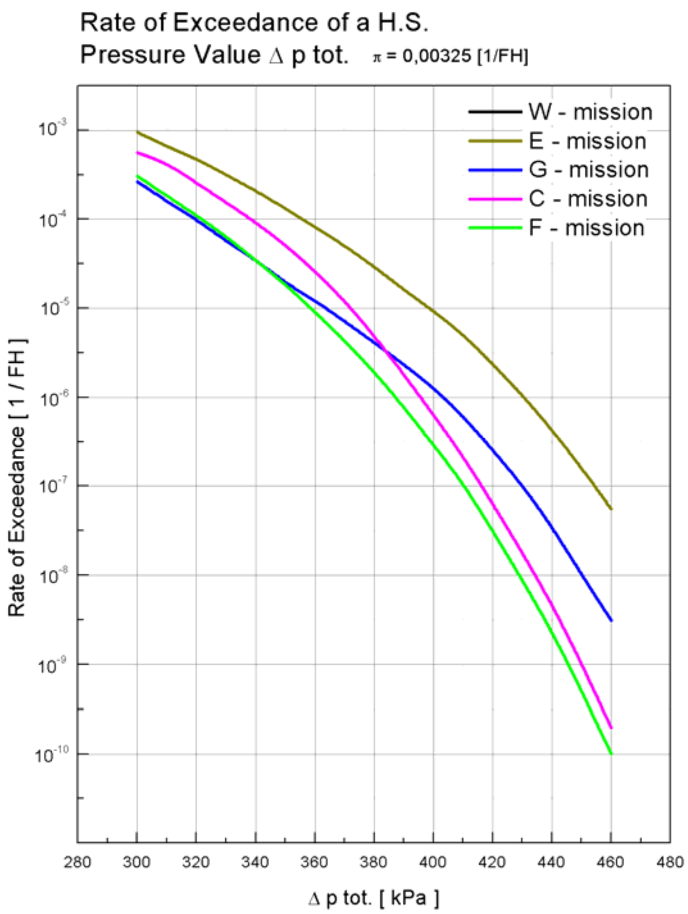


Fig. 20 Rate of exeedance of Hammershock pressure

Total H.S. pressure at AIP [kPa]	No of occurrence of H.S. pressure in given range per A/C fleet life				
	C mission	E mission	G mission	F mission	W mission
$300 \le p < 325$	1751.27000	2639.57000	1000.05000	986.39000	2407.80000
$325 \le p < 350$	789.94000	1270.96000	292.87000	319.12000	1269.35000
$350 \le p < 375$	213.04000	445.23000	69.86000	76.34000	471.38000
$375 \le p < 400$	33.63400	143.77000	20.99200	12.87000	148.48000
$400 \le p < 425$	3.08700	39.43000	5.41500	1.44200	39.67000
$425 \le p < 450$	0.17220	7.31500	0.76890	0.08960	7.31800
$450 \le p < 475$	0.00512	0.77700	0.05136	0.00266	0.77770
$475 \le p < 500$	0.00007	0.04280	0.00155	0.00003	0.04284
$500 \le p < 525$	0.00000	0.00116	0.00002	0.00000	0.00116
$525 \le p < 550$	0.00000	0.00002	0.00000	0.00000	0.00002

Table 3 Number of occurrences of Hammershock

Some results are given in Fig. 19, Table 2 and Fig. 20, Table 3. In detail Fig. 19 gives the probability density function $f_{\Delta ptot}(p)$ for the E mission speed spectrum according to equation 26, Table 2 gives the rate of occurrence of Hammershock pressure levels per flying hour according to equation 1 and 30. Fig. 20 shows the rate of exceedance of Hammershock pressure per flying hour according to equation 2 and 32. Table 3 depicts the number of occurrences of Hammershock pressure levels $(a < \Delta p_{tot} \leq b)$ according to the equation

$$N(a,b) = TF \cdot \lambda(a,b) \tag{34}$$

Remark: The total expected number N of Hammershock events per A/C fleet is given by

$$N = TF \cdot \lambda \tag{35}$$

i.e. in this case $N = 4.8 \cdot 10^6 FH \cdot 3 \cdot 10^{-3} \cdot \frac{1}{FH} = 14400.$

4.1.1 Statistical Simulation by means of Monte Carlo Simulation

Another approach has been chosen in order to compare the results. This approach is known as Monte Carlo method. (Ref. 4)
The essence of the Monte Carlo Simulation is fast explained. By means of a pseudo random number generator (representing a uniformly distributed random variable over the interval [0, 1] one generates by suitable methods numbers ξ_i, v_i, t_i which represent respectively the Hammershock intensity ξ, the speed v and the temperature T (i.e. cold day, isa day or hot day). Obviously the generated numbers ξ_i, v_i, t_i have to fulfill all the properties of these random variables, for example the numbers ξ_i must be normally distributed with $\mu = 0$ and $\sigma = 1$ etc.

With these data (intensity ξ_i, speed v_i and “day” t_i) and choosing the suitable equation from (23), the corresponding pressure can be calculated. Repeating this procedure n times one gets a considerable pressure sample from which one can derive a relative frequency distribution. Multiplying this relative frequency distribution by the expected number N of Hammershock per A/C fleet life an approximation for the expected frequency distribution of the Hammershock can be obtained.

The theoretical basis of this procedure is the well known “weak” law of large numbers of BERNOULLI.
According to this theorem the relative frequency of an event A tends in probability to the probability of the event A if the number of trials $n \to \infty$ and if the trials are stochastically independent.

Therefore

$$prob(A) \sim \frac{n(A)}{n}$$

where n(A) denotes the number of occurrences of the event A among n independent trials.

A Monte Carlo simulation for the five different speed spectra (n=300 000) has been made obtaining the results shown in Table 4.

A comparison of Table 3 with Table 4 shows a very good agreement of the results obtained by different methods and suggests the correctness of both methods.

Total H.S. pressure at AIP [kPa]	No of occurrence of H.S. pressure in given range per A/C fleet life				
	C mission	E mission	G mission	F mission	W mission
$300 \le p < 325$	1733.20000	2642.23000	1002.53000	982.18000	2409.30000
$325 \le p < 350$	783.43000	1275.01000	2894.05000	317.40000	1276.55000
$350 \le p < 375$	218.25000	451.95000	66.31000	76.52000	466.07000
$375 \le p < 400$	33.15000	142.11000	19.51000	12.68000	145.57000
$400 \le p < 425$	2.50300	38.26000	5.51000	1.44200	41.90000
$425 \le p < 450$	0.18314	7.70000	0.67300	0.04580	6.54500
$450 \le p < 475$	0.00000	0.67000	0.12200	0.00000	0.48930
$475 \le p < 500$	0.00000	0.00000	0.00000	0.00000	0.06116
$500 \le p < 525$	0.00000	0.00000	0.00000	0.00000	0.00000
$525 \le p < 550$	0.00000	0.00000	0.00000	0.00000	0.00000

Table 4 Number of occurrence of Hammershock pressure

4.2 Definition of Hammershock design pressure and results

Three different approaches have been chosen:

- In order to gets an overall impression of the results the value $\Delta p_{tot-\max} = 449\,\text{kPa}$ is taken which covering the subsonic, transonic and supersonic design range. The number of exceedances of this Hammershock pressure per A/C fleet life is:

	Number of exceedances per A/C fleet life $\left(= 4.8 \cdot 10^6\, FH\right), \quad \lambda = 3 \cdot 10^{-3} \cdot \dfrac{1}{FH}$				
$\Delta p_{tot} \quad [kPa]$	C mission	E mission	G mission	F mission	W mission
$\Delta p_{tot} \ge 449 \quad [kPa]$	0.0061	0.911	0.06	0.00315	0.911

(36)

- A first improvement is made by subdividing between max. subsonic $(Ma \le 0.9)$ and supersonic $(Ma \ge 1.02)$ pressures to be superimposed to the respective subsonic/transonic/supersonic flight cases in practical design:

a) $\Delta p_{tot-\max} = 408. \quad [kPa] \quad for \quad Ma \le 0.9$

b) $\Delta p_{tot-\max} = 408. \quad [kPa] \quad for \quad 0.9 < Ma < 1.02$

c) $\Delta p_{tot-\max} = 449. \quad [kPa] \quad for \quad Ma \ge 1.02$

The number of exceedances per A/C fleet life of these three ranges can be calculated by multiplying TF by $\lambda \cdot p_{a,\infty,v_a,v_e}$ according to equation (31), where

Event	$a[kPA]$	$v_a[Kts]$	$v_e[Kts]$
a	408.	0.	595.34
b	408.	595.34	674,72
c	449.	674.72	750.0

The results are

	Number of exceedances per A/C fleet life $(= 4.8 \cdot 10^6\, FH)$, $\lambda = 3 \cdot 10^{-3} \cdot \frac{1}{FH}$				
event	C mission	E mission	G mission	F mission	W mission
a	0.414	1.110	0.177	0.179	1.180
b	0.944	8.574	1.373	0.49	8.575
c	0.0	0.831	0.045	0.	0.831
Total	1.358	10.515	1.595	0.669	10.586

(37)

- In order to finally consider proper assignment of transonic pressures and flight conditions a further improvement is introduced by Δp_{tot} at Ma=0.95:

a) $\Delta p_{tot-\max} = 408. \quad [kPA] \quad for \quad Ma \le 0.9$

b) $\Delta p_{tot-\max} = 425. \quad [kPA] \quad for \quad 0.9 < Ma < 1.02$

c) $\Delta p_{tot-\max} = 449. \quad [kPA] \quad for \quad Ma \ge 1.02$

This leads to the following number of exceedances of these Hammershock pressures

	Number of exceedances per A/C fleet life $(= 4.8 \cdot 10^6\, FH)$, $\lambda = 3 \cdot 10^{-3} \cdot \frac{1}{FH}$				
event	C missions	E missions	G missions	F missions	W missions
a	0.414	1.110	0.177	0.179	1.180
b	0.151	1.571	0.269	0.077	1.571
c	0.0	0.831	0.045	0.	0.831
Total	0.565	3.512	0.491	0.256	3.582

(38)

4.3 Refinement of the assumptions defined in 4.1

In order to get an impression on the sensitivity of assumptions the following data have been varied in accuracy:

- Temperature
    - No change

- Frequency of Hammershock
    - According to combat Aircraft experience Fig. 21 has been considered. An evaluation of the area under the curve shown in Fig. 21, depicts that the total number of Hammershock events during 60 000 engine hours which is 30 000 flight hours (two engine A/C). This number is about 97.5 which means a constant rate of Hammershock occurrence of

$$\lambda = \frac{97.5}{30000}\frac{1}{FH} = \frac{3.25}{1000}\frac{1}{FH}$$

Therefore this value for the improved analysis has been used.

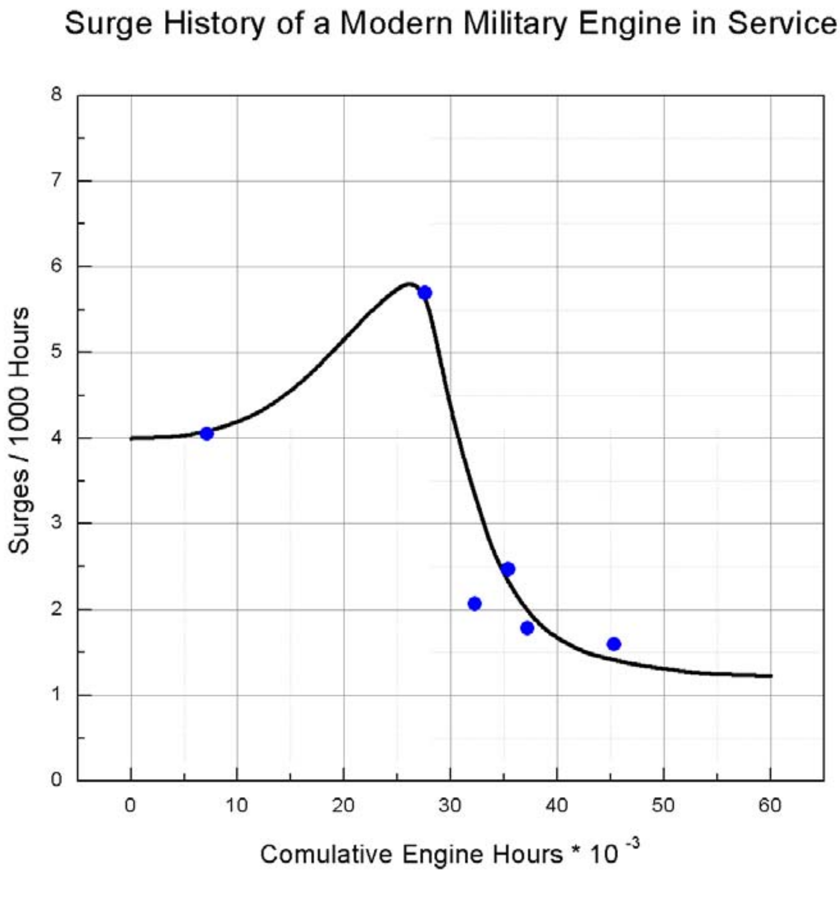


Fig. 21 Surge History of modern military engine in service

- Aircraft fleet life TF
    - No change

- Total design Hammershock pressure vs. speed at sea level
    - According to pressure assumptions (see Fig. 13 – 16) this dependence is not linear as assumed in prior analysis. A better approximation (stepwise linear function) of these functions has been made as follows, see Fig. 22.

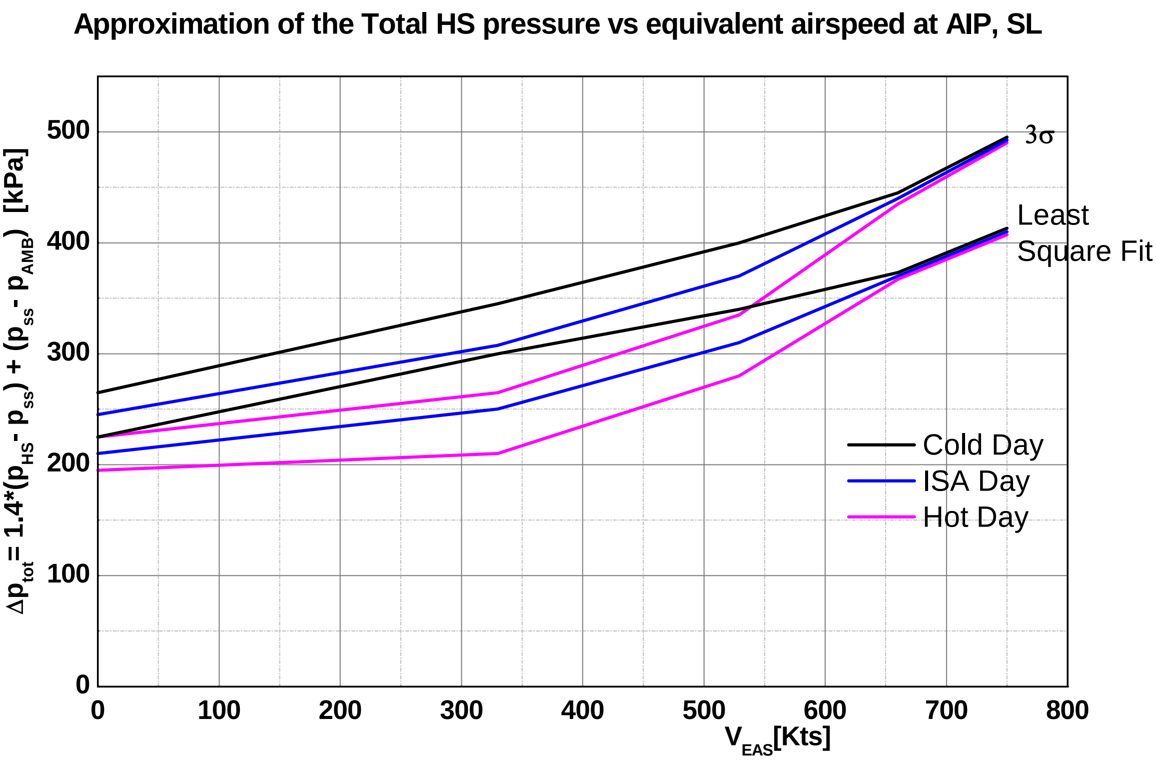


Fig. 22 Approximation of the total Hammershock pressure

Day i	$v_{EAS}\,[kts]$	Δp_{tot} [kPa]	
		least square fit	$3\cdot\sigma$
1 = cold	$0\le v\le 330$ $330\le v\le 529$ $529\le v\le 660$ $660\le v\le 750$	$f_{01}(v)=\begin{cases}225+\frac{5}{22}v\\300+\frac{40}{199}(v-330)\\340+\frac{30}{131}(v-529)\\370+\frac{37}{90}(v-660)\end{cases}$	$f_{11}(v)=\begin{cases}265+\frac{8}{33}v\\345+\frac{57}{199}(v-330)\\402+\frac{43}{131}(v-529)\\445+\frac{43}{90}(v-660)\end{cases}$
2 = isa	$0\le v\le 330$ $330\le v\le 529$ $529\le v\le 660$ $660\le v\le 750$	$f_{02}(v)=\begin{cases}210+\frac{3}{22}v\\255+\frac{55}{199}(v-330)\\310+\frac{62}{131}(v-529)\\372+\frac{33}{90}(v-660)\end{cases}$	$f_{12}(v)=\begin{cases}245+\frac{4}{22}v\\305+\frac{65}{199}(v-330)\\370+\frac{70}{131}(v-529)\\440+\frac{42}{90}(v-660)\end{cases}$

(20')

Hence

Day i	$v_{EAS}[kts]$	Δp_{tot} [kPa]	
		least square fit	$3 \cdot \sigma$
3 = hot	$0 \le v \le 330$ $330 \le v \le 529$ $529 \le v \le 660$ $660 \le v \le 750$	$f_{03}(v) = \begin{cases} 195 + \frac{1}{22} v \\ 210 + \frac{70}{199}(v-330) \\ 280 + \frac{84}{131}(v-529) \\ 375 + \frac{29}{90}(v-660) \end{cases}$	$f_{13}(v) = \begin{cases} 225 + \frac{4}{33} v \\ 265 + \frac{73}{199}(v-330) \\ 338 + \frac{97}{131}(v-529) \\ 435 + \frac{41}{90}(v-660) \end{cases}$

(20')

Inserting these functions in (3) similar expressions for $\Delta p_{tot(i)}$ with different coefficients have been derived.

- Hammershock intensity ξ
  - No change

- Speed spectra v(t)
  - The w mission case has been changed by taking the greatest values from 400. Kts on and adjusting the remaining value to get a sum of 60 minutes (see Table 5). Hence $\mu = 434.88\ [Kts]$ and $\sigma = 91.015\ [Kts]$

V_{EAS} [kts]	minutes pro hour spent in a given speed range				
	C mission	E mission	G mission	F mission	W mission
$0 \le v < 50$	0.100	0.150	0.030	0.100	0.100
$50 \le v < 100$	0.200	0.200	0.130	0.200	0.200
$100 \le v < 150$	0.300	0.300	0.180	0.600	0.300
$150 \le v < 200$	3.300	1.000	3.640	1.400	0.700
$200 \le v < 250$	5.000	1.500	1.850	3.500	1.200
$250 \le v < 300$	8.900	2.200	19.160	10.000	1.500
$300 \le v < 350$	1.600	2.700	2.462	25.000	2.500
$350 \le v < 400$	17.100	3.300	22.300	7.800	3.150
$400 \le v < 450$	12.800	30.000	6.470	6.200	30.000
$450 \le v < 500$	3.500	10.000	2.880	3.800	10.000
$500 \le v < 550$	6.700	5.000	0.331	1.000	6.700
$550 \le v < 600$	0.300	2.000	0.358	0.300	2.000
$600 \le v < 650$	0.200	1.000	0.084	0.100	1.000
$650 \le v < 700$	0.000	0.500	0.125	0.000	0.500
$700 \le v < 750$	0.000	0.150	0.000	0.000	0.150

Table 5 Time spend in a given speed range (mins/hours)

Usually safety margin analysis is based on specific missions. A comparison of known definitions for a modern fighter A/C (as given in Table below) shows that these missions are fully covered by the E mission spectrum and W mission spectrum

No	Missions	Equivalent to Mission	Flight time [min]	Frequency [%]
1	Intercept CAP High Altitude	G mission / 3	90	23
2	Med/Low Level Interception	E mission / 1	< 105	46
3	Low Level Navigation	G mission / 6	90	17
4	Ground Attack	E mission / 3	< 105	14

4.3.1 Consequences due to Modification

The consequences of these modifications are

a) joint probability density of $\Delta p_{tot(i)}$ and v

$$f_{\Delta ptot(i),v}(p,v)=\sum_{j=1}^{4}\chi_{\{v_{j-1}\le v\le v_j\}}\cdot\frac{f_v(v)}{|\gamma_{ij}+\delta_{ij}\cdot v|}\cdot f_\xi\left(\frac{p-\alpha_{ij}-\beta_{ij}\cdot v}{\gamma_{ij}+\delta_{ij}\cdot v}\right) \qquad (11')$$

b) probability density of $\Delta p_{tot(i)}$

$$f_{\Delta ptot(i)}(p)=\sum_{j=1}^{4}\int_{v_{j-1}}^{v_j}\frac{f_v(v)}{|\gamma_{ij}+\delta_{ij}\cdot v|}\cdot f_\xi\left(\frac{p-\alpha_{ij}-\beta_{ij}\cdot v}{\gamma_{ij}+\delta_{ij}\cdot v}\right)\cdot dv \qquad (12')$$

c) joint probability density of Δp_{tot} and v

$$\begin{aligned} f_{\Delta ptot,v}(p,v) &= \sum_{i=1}^{3}p_i\cdot f_{\Delta ptot(i),v}(p,v)= \\ &=\sum_{j=1}^{4}\chi_{\{vj-1\le v\le vj\}}\cdot\sum_{i=1}^{3}p_i\frac{f_v(v)}{|\gamma_{ij}+\delta_{ij}\cdot v|}\cdot f_\xi\left(\frac{p-\alpha_{ij}-\beta_{ij}\cdot v}{\gamma_{ij}+\delta_{ij}\cdot v}\right)\end{aligned} \qquad (25')$$

d) probability density of Δp_{tot}

$$f_{\Delta ptot}(p)=\sum_{j=1}^{4}\int_{v_{j-1}}^{v_j}\sum_{i=1}^{3}p_i\cdot\frac{f_v(v)}{|\gamma_{ij}+\delta_{ij}\cdot v|}\cdot f_\xi\left(\frac{p-\alpha_{ij}-\beta_{ij}\cdot v}{\gamma_{ij}+\delta_{ij}\cdot v}\right)\cdot dv \qquad (26')$$

where $v_0=0.\ [Kts]$, $v_1=330.\ [Kts]$, $v_2=529.\ [Kts]$, $v_3=660.\ [Kts]$, $v_4=750.\ [Kts]$ and

$$\chi\{v_{j-1} \le v \le v_j\} \quad (v) = \begin{cases} 1 & for \quad v_{j-1} \le v \le v_j \\ 0 & \text{otherwise} \end{cases}$$

is the so called characteristic function of $\{v_{j-1} \le v \le v_j\}$. The resulting formulae are:

$$P_{a,b,v_a,v_e} = \sum_{j=1}^{4} \int_{v_{j-1}}^{v_j} f_v(v) \cdot \chi_{\{v_a \le v \le v_e\}} \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \left[erf\left(\frac{a - \alpha_{ij} - \beta_{ij} \cdot v}{\sqrt{2}(\gamma_{ij} + \delta_{ij} \cdot v)} \right) - erf\left(\frac{b - \alpha_{ij} - \beta_{ij} \cdot v}{\sqrt{2}(\gamma_{ij} + \delta_{ij} \cdot v)} \right) \right] \right) \cdot dv \tag{29'}$$

$$P_{a,b} = \sum_{j=1}^{4} \int_{v_{j-1}}^{v_j} f_v(v) \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \left[erf\left(\frac{b - \alpha_{ij} - \beta_{ij} \cdot v}{\sqrt{2}(\gamma_{ij} + \delta_{ij} \cdot v)} \right) - erf\left(\frac{a - \alpha_{ij} - \beta_{ij} \cdot v}{\sqrt{2}(\gamma_{ij} + \delta_{ij} \cdot v)} \right) \right] \right) \cdot dv \tag{30'}$$

$$P_{a,\infty,v_a,v_e} = \sum_{j=1}^{4} \int_{v_{j-1}}^{v_j} f_v(v) \cdot \chi_{\{v_a \le v \le v_e\}} \cdot \left(\sum_{i=1}^{3} \frac{P_i}{2} \cdot erfc\left(\frac{a - \alpha_{ij} - \beta_{ij} \cdot v}{\sqrt{2}(\gamma_{ij} + \delta_{ij} \cdot v)} \right) \right) \cdot dv \tag{31'}$$

$$P_{a,\infty} = \sum_{j=1}^{4} \int_{v_{j-1}}^{v_j} f_v(v) \cdot \left(\sum_{i=1}^{3} \frac{p_i}{2} \cdot erfc\left(\frac{a - \alpha_{ij} - \beta_{ij} \cdot v}{\sqrt{2}(\gamma_{ij} + \delta_{ij} \cdot v)} \right) \right) \cdot dv \tag{32'}$$

The results of this modified analysis are shown in Table 6, Fig. 23 and Table 7.

Total H.S. pressure at AIP [kPa]	Rate of occurrence of H.S. pressure in given range per flying hour [1 / FH]				
	C mission	E mission	G mission	F mission	W mission
$300 \le p < 325$	3.788E-04	5.206E-04	2.591E-04	2.671E-04	5.466E-04
$325 \le p < 350$	1.520E-04	2.515E-04	7.748E-05	7.480E-05	2.673E-04
$350 \le p < 375$	3.487E-05	8.413E-05	1.480E-05	1.448E-05	8.870E-05
$375 \le p < 400$	5.023E-06	2.995E-05	4.262E-06	2.065E-06	2.872E-05
$400 \le p < 425$	4.067E-07	8.468E-06	1.228E-06	1.898E-07	8.495E-06
$425 \le p < 450$	1.717E-08	1.837E-06	1.784E-07	8.782E-09	1.838E-06
$450 \le p < 475$	3.522E-10	2.511E-07	1.126E-08	1.819E-10	2.511E-07
$475 \le p < 500$	3.271E-12	1.966E-08	3.077E-10	1.633E-12	1.966E-08
$500 \le p < 525$	1.307E-14	8.038E-10	3.755E-12	6.572E-15	8.038E-10
$525 \le p < 550$	2.123E-17	1.624E-11	2.120E-14	1.063E-17	1.624E-12

Table 6 Rate of occurrence of Hammershock pressure

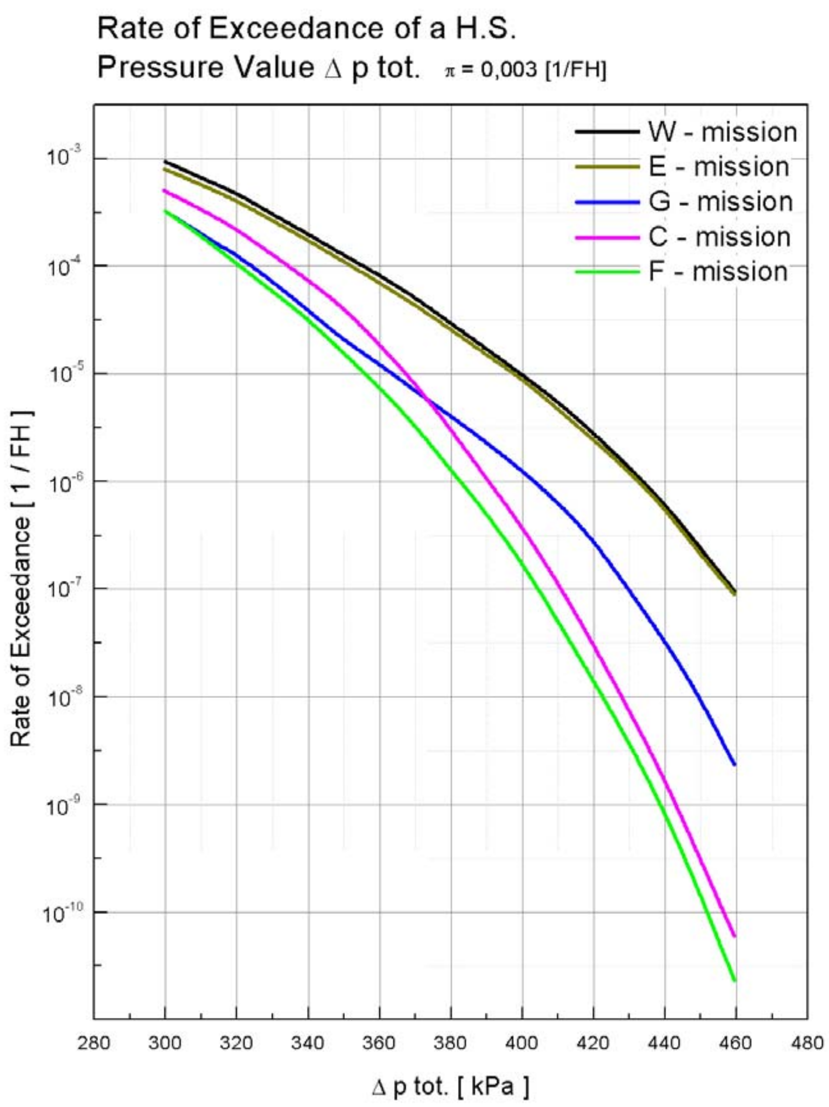


Fig. 23 Rate of Exceedence of Hammershock

Total H.S. pressure at AIP [kPa]	No of occurrence of H.S. pressure in given range per A/C fleet life				
	C mission	E mission	G mission	F mission	W mission
$300 \le p < 325$	1818.17000	2498.74000	1243.45000	1282.16000	2623.47000
$325 \le p < 350$	729.70000	1207.43000	371.91000	359.06000	1283.16000
$350 \le p < 375$	167.38000	403.82000	71.02000	69.51000	426.57000
$375 \le p < 400$	24.11000	132.76000	20.46000	9.91300	135.71000
$400 \le p < 425$	1.95200	40.65000	5.90000	0.91100	40.77000
$425 \le p < 450$	0.08240	8.82000	0.85600	0.04220	8.82000
$450 \le p < 475$	0.00017	1.20500	0.05400	0.00870	1.20500
$475 \le p < 500$	0.00002	0.09440	0.00148	0.00001	0.09440
$500 \le p < 525$	0.00000	0.00386	0.00002	0.00000	0.00386
$525 \le p < 550$	0.00000	0.00008	0.00000	0.00000	0.00008

Table 7 Number of occurrences of hammershock pressure

By applying the same approach as done in chapter 4.2 the modifications show very small influences to the original tables 36 to 38 as follows respectively:

1a) $\Delta p_{tot-\max} = 449.\ [kPa]$

event	Number of exceedances per A/C fleet life $(= 4.8 \cdot 10^6\ FH)$, $\lambda = 3.25 \cdot 10^{-3} \cdot \frac{1}{FH}$				
	C missions	E missions	G missions	F missions	W missions
a	0.002	1.43	0.063	0.0011	1.43

(36')

2a) $\Delta p_{tot-\max} = 408.\ [kPa]$ *for* $Ma \leq 0.9$

b) $\Delta p_{tot-\max} = 408.\ [kPa]$ *for* $0.9 \leq Ma \leq 1.02$

c) $\Delta p_{tot-\max} = 449.\ [kPa]$ *for* $Ma \geq 1.02$

event	Number of exceedances per A/C fleet life $(= 4.8 \cdot 10^6\ FH)$, $\lambda = 3.25 \cdot 10^{-3} \cdot \frac{1}{FH}$				
	C mission	E mission	G mission	F mission	W mission
a	0.2045	0.505	0.0796	0.0836	0.541
b	0.5849	6.934	1.2543	0.3005	6.938
c	0.0	1.386	0.0546	0.0	1.386
Total	0.7894	8.825	1.3885	0.3841	8.865

(37')

3a) $\Delta p_{tot-\max} = 408.\ [kPa]$ *for* $Ma \leq 0.9$

b) $\Delta p_{tot-\max} = 425.\ [kPa]$ *for* $0.9 \leq Ma \leq 1.02$

c) $\Delta p_{tot-\max} = 449.\ [kPa]$ *for* $Ma \geq 1.02$

event	Number of exceedances per A/C fleet life $(= 4.8 \cdot 10^6\ FH)$, $\lambda = 3.25 \cdot 10^{-3} \cdot \frac{1}{FH}$				
	C missions	E missions	G missions	F missions	W missions
a	0.2045	0.505	0.0796	0.0836	0.541
b	0.0730	1.093	0.2131	0.0373	1.093
c	0.0	1.386	0.0546	0.0	1.386
Total	0.2775	2.984	0.3473	0.1209	3.020

(38')

Moreover, the expected total number of Hammershock events is now:

$$N = 4.8 \cdot 10^{6}\, FH \cdot 3.25 \cdot 10^{-3} \frac{1}{FH} = 15600.$$

5. Conclusions Probabilistic

The results obtained in chapter 4 show that the expected number of exceedances of the Hammershock design values (case 3) is about 3.5 respectively 3. in the revised version of the example, or in other words, the rate of exceedances is about $7.3 \cdot 10^{-7}\ {}^{1}\!/_{FH}$, respectively $6.25 \cdot 10^{-7}\ {}^{1}\!/_{FH}$.

The refinement in 4.3 did not change very essentially the results; see for comparison Table 3 and Table 6.
It must be emphasized that this analysis, for both assumptions, has been restricted to sea level, since the Hammershock pressure as a function of the equivalent airspeed is almost constant over the altitude H. A more precise analysis can be carried out if instead of the speed spectrum a joint altitude Mach Number spectrum as in Fig. 18 is available. The results, however, are expected to be less critical than those obtained in this analysis, since the Hammershock design pressure decreases more rapidly with the Ma-number than with the equivalent airspeed.

Finally, it is knows that most Hammershock events occur during flight maneuvers, so for structure design it is very important to consider the combination of Hammershock events and coincident normal load factor n_z . This will be discussed in a second paper.

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7. Notation

AIP	Aerodynamic Interface Plane
Anti-	Anti-Symmetric
BPR	By-Pass Ratio
F/F	Front Fuselage
GSIT	Ground Surge Interaction Trial
Hz	Hertz [1/sec]
kPa	Kilo-Pascal (10^3 [N/m^2])
i	i = 1 cold day i = 2 isa day i = 3 hot day
H.S.	Hammershock
Nz	Normal Aircraft Acceleration - Normal Load Factor
N_H	Engine setting
O/P	Overpressure
OPR	Over Pressure Ratio p_{1max}/p_{1ss}
p_{HS}	hammershock pressure
p_{SS}	steady state pressure
p_{AMB}	ambient pressure
Δp_{tot}	$= 1.4 \cdot (p_{HS} - p_{ss}) + (p_{ss} - p_{AMB})$ total H.S. pressure atAIP
$\Delta p_{tot(i)}$	$= 1.4 \cdot (p_{HS} - p_{ss}) + (p_{ss} - p_{AMB})$ for day i
T	random variable temperature
v	random variable equivalent airspeed
ξ	random variable intensity of surge
$f_T(x)$	probability density T
$f_v(x)$	probability density v
$f_\xi(x)$	probability density ξ
$f_{0i}(v)$	total hammershock overpressure at AIP as a function of v (least square fit)
$f_{1i}(v)$	total hammershock overpressure at AIP as a function of ($3 \cdot \sigma$)
α_{0i}	= total hammershock overpressure (least square fit) at $v_e = 750$ kts
α_{1i}	= total hammershock overpressure ($3 \cdot \sigma$) at $v_e = 750$ kts
β_{0i}	$= \dfrac{dp_{tot(i)}}{dv}$ slope of the total hammershock overpressure (least square fit)
β_{1i}	$= \dfrac{dp_{tot(i)}}{dv}$ slope of the total hammershock overpressure ($3 \cdot \sigma$)
$f_{\Delta ptot(i)}(p)$	probability density of $\Delta p_{tot(i)}$
$f_{\Delta ptot(i),v}(p)$	joint probability density of $\Delta p_{tot(i)}$ and v

$f_{\Delta ptot}(p)$	probability density of Δp_{tot}
$f_{\Delta ptot,v}(p,v)$	joint probability density of Δp_{tot} and v
$F(p)$	distribution function of Δp_{tot}
$F(p,v)$	joint probability density of Δp_{tot} and v
$P_{a,b}$	probability that $\Delta p_{tot} \geq a$ and $\Delta p_{tot} \leq b$
P_{a,b,v_a,v_e}	probability that $a \leq \Delta p_{tot} \leq b$ and $v_a \leq v_{EAS} \leq v_e$
TF	A/C fleet life
λ	frequency of hammershock
$N = TF \cdot \lambda$	expected number of H.S. per A/C fleet life
$N(a,b) = TF \cdot \lambda(a,b)$	expected number of H.S. occurences (per A/C fleet life) with pressure between a and b
$\lambda(a,b) = \lambda \cdot p_{a,b}$	rate of occurrence of H.S. per A/C flight hour with a pressure value between a and b
$\lambda(a,b,v_a,v_b)$	$= \lambda \cdot P_{a,b,v_a,v_e}$
erf(x)	error function
erfc(x)	complementary error function
sign(x)	sign function
$\psi(x)$	distribution function
$\chi_A(x)$	characteristic function of A
$\sigma(x)$	unit step function
$\delta(x)$	DIRAC distribution (impulse function)
μ	mean value
σ	standard deviation